

# Mathematics teacher candidates' approaches to using topology in geometry

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## Abstract

When we look at the theoretical relationships between the three types of geometry, we find that the most primitive type is topology and that both Euclidean and projective geometry are derived from this earlier type. Therefore, Piaget described the first stage or level of understanding geometry as topological understanding. In this study, the geometric thinking conditions of mathematics teacher candidates were examined and what they think about the topology course was determined by surveying their opinions about the undergraduate courses. The participants of the study consisted of 36 teacher candidates studying in the Mathematics Department of the Faculty of Science attached to a university in Turkey. The findings of the study revealed that none of the teacher candidates used topological thinking to solve the questions in the data collection instrument. Those candidates who considered topology as an abstract course did not associate topology with geometry. A thorough understanding of the meaning of topology and topological thought is an advantage to young children as well as college-age students; improving mathematics curricula on this topic, particularly at the graduate level, will ensure that all future teachers are appropriately trained for their future responsibility of educating others. We believe that it is necessary to give Topology and topological thought the place they deserve in the educational process.

## KEYWORDS

Euclidean geometry, teacher candidate, topology, topological geometry

## 1 | INTRODUCTION

Geometry is a branch of mathematics that deals with the study of space and shapes that can be designed in space or the subbranch of mathematics dealing with spatial relationships. The origins of the study of geometry date back to 3,000 BC. Etymologically, the word “geometry” consists of two root terms, *geo* and *metry*, which can be translated as “location” and “measurement,” respectively. It is thought that Thales (624–547 BC) and Pythagoras

(569–475 BC) were the first to identify the basic rules of geometry and to use the method of proof were (Doğan, 2013). Euclid (325–265 BC) later adapted the method of proof to geometry and wrote *Elements*. Based on this, the teaching and use of geometry implemented Euclidean postulates and proofs. Euclidean geometry, despite being confined to metric features, remained the primary system of geometry until the eighteenth century, when theorists began to pay more attention to the nonmetric features of shapes (Delice & Karaaslan, 2016). Centuries later, in a

1741 paper titled “Solutio problematis ad geometriam situs pertinentis” (“The solution to a problem relating to the geometry of position”) Euler presented a new theory of geometry that did not rely on the concept of distance and measure but on relative positions (Karaca, 2013). By the mid-nineteenth century, independent of Euclidean geometry, Euler's work had taken its place in the field as the new geometry systems of projective geometry, affine geometry, and topology (Hacısalıhoğlu, 1975, pp. 1–16). The famous French mathematician Henri Poincaré (1854–1912) explained topology as a discipline that enables us to learn geometric shapes, not only in customary space, but also in three-dimensional spaces.

## 1.1 | Theoretical framework

Topology is generally defined as “space science.” It is derived from *topos*, meaning surface or place, and *logos*, meaning science. Topology is the practice of measuring and examining new shapes created by bending, stretching, tightening, or otherwise altering known geometric shapes using topology. The cutting, tearing, and shredding of shapes are techniques employed in topological thinking: Transformations accepted in topology include elastic movements such as pulling, compressing, displacing, rotating, bending, and stretching shapes.

In his work on the understanding of geometry levels, Piaget expressed the levels of understanding geometry as Topological Understanding, Projective Understanding, and Euclidean Understanding. A joint work by Piaget and Inhelder (1967) found that

representational thought or imagination at first appears to ignore metric and perspective relationships, proportions, etc. Consequently, it is forced to reconstruct space from the most primitive notions such as the topological relationships of proximity, separation, order, enclosure, etc., applying them to metric and projective figures yielded by perception. (p. 4)

Piaget described the first stage of understanding geometry as topological understanding, which is the idea that shapes in topological spaces (“rubber-sheet geometry”) are not as constant and invariant as they are in Euclidean space. According to Piaget, a child understands his or her environment topologically before doing so in a Euclidean sense, because nothing is fixed or unchanging. He felt that children first learn to easily recognize the shapes of objects in their surroundings and recognize the topological properties of shapes through the activities they perform. Later, around the age of 3 years, they begin to recognize and distinguish between Euclidean shapes such as triangles, rectangles, and squares (Piaget & Inhelder, 1967).

Van Hiele's theory of the development of geometric thinking later emerged as an elaboration of Piaget's approach to the development of geometric thinking in children. According to Van Hiele (1986), the development of geometric thinking consists of five levels. Altun (2013) explained these levels as follows:

First level (visualization): At this level, students can recognize geometric objects and explain what they look like. Children at this level perceive geometric shapes and objects as a whole.

Second level (analysis): In this phase, children think of all shapes as having the same characteristics instead of as geometric shapes, and count all the features of a given shape. They can, however, make some generalizations about geometric shapes.

Third level (abstraction; prior to logical or experience-related deduction): In the logical inference phase, children are able to compare features related to shapes. Children at this level develop the ability to make connections between shape categories.

Fourth level (deduction; logical inference): In this phase, students will have gained the ability to use proofs. Individuals at this level can create proofs themselves and see the applications of a theorem.

Fifth level (rigor; top level): Students may interpret the axioms, theorems, and definitions of Euclidean geometry in non-Euclidean geometries and perform applications. Individuals can study this level of geometry as a science and conduct academic research.

Van Hiele believe that progress from one level to the next level is more dependent on educational experiences than on age or maturation (Mason, 1998). However, Van Hiele stated that Piaget's theory of geometric thinking was a theory of development, not a theory of education (Van Hiele, 1986). Piaget's levels of understanding are associated with biological development, whereas Van Hiele's levels of understanding depend on the training provided (Aslan & Arnas, 2007).

The geometric thinking approach that directs this study is the topological understanding approach expressed by Piaget. Piaget explained that children first perceive their environment and shapes at an early age with topological geometry; therefore, it is particularly appropriate to start teaching geometry to children using topological geometry rather than Euclidean geometry (Piaget & Inhelder, 1967). Topological geometry is more comprehensive than Euclidean geometry, as it includes affine geometry, projective geometry, and Euclidean geometry.

Topology is the common source for the other two types (Euclidean and projective) of geometry. It is an interesting question, then, whether in the development of thinking in children geometry follows the historic order or the theoretical order. More precisely, will we find that Euclidean intuitions and operations develop first, and topological

intuitions and operations later? Or will we find that the relationship is the other way around? What we do find, in fact, is that the first intuitions are topological (Piaget, 1971, pp. 30–31).

Some studies have paid attention to the integration of topology and geometric information (Clementini & Di Felice, 1996; Decuypere & Simons, 2016; Freitas & McCarthy, 2014; Martin & Secor, 2014). Topology may be described as “rubber sheet” geometry (Arnold, 1963). According to Turro (1986), this definition emphasizes the elasticity of the concepts of topology, which is concerned, in general, only with very fundamental geometric properties. Topological properties may be visualized as those geometric properties of a figure on a rubber sheet that are conserved upon twisting and stretching, such as the connectivity, the sequence, and the continuity of points. Freitas and McCarthy (2014) argue that experiences with topology allow middle school students to develop a more robust understanding of orientation and dimension. Topology, which contains sets and geometry, is thought to fit the approach of the newly developed mathematics curriculum in that it helps one to see the relations between the different geometries, to look at geometric relations from a different perspective, and to reflect on the area of mathematics developed in teaching in this field by reducing it to a simple level (Delice & Karaaslan, 2016). Changing and developing students’ geometric thinking approaches is closely related to the education they receive. Therefore, teachers and teacher candidates have a great responsibility in this regard. The more detailed framework of geometric thought that is expected to emerge from the research performed will contribute significantly to how students are taught to think about geometry. More importantly, this framework will help teachers guide their students in connecting with other ways of thinking, allowing them to conceptualize geometry-related concepts in more detail and in various ways (Greenstein, 2014).

## 1.2 | Purpose of the study

Topology is a subject that students are rarely exposed to before the undergraduate level; it is often reserved for graduate level students. This research was intended to draw attention to the place of topology in education and to examine the relationship between topological thinking and geometry. In addition, the teacher candidates’ geometric thinking abilities were also investigated. Geometric thinking abilities of senior teacher candidates in Turkey were assessed in the context of Piaget’s topological understanding framework. The researchers also sought to determine what the teacher candidates thought about the topology course by surveying their ideas

about undergraduate courses. We believed that the study will contribute to mathematics education, given that there are few studies of this topic in the national and international arena.

## 2 | METHOD

The aim of this study was to examine the thoughts and opinions of the teacher candidates regarding topology and topological thought in detail. It is stated that the topology will enable to see the relationships between different geometries and to provide a different view of geometric relations (Delice & Karaaslan, 2016; Greenstein, 2014). To accomplish this aim, the following research questions were answered:

- What is the importance of topology in education?
- To what extent are teacher candidates aware of the relationship between topological thinking and geometry?
- How do teacher candidates define the content of topology course? To what extent are they aware of the relationship between the topology course and geometry?

To answer these research questions, qualitative methodology was chosen. The data of this qualitative study included the teacher candidates written responses to four open-ended questions and semi-structured written opinions of their views on undergraduate courses and their reflections on secondary education.

### 2.1 | Study group

The participants of the study comprised of 36 senior teacher candidates studying at a Mathematics Department and receiving formal pedagogical training who will be teaching at high schools. Eighteen of the participants (50%) were female and 18 (50%) were male. Criterion sampling, which provides opportunities to work with people, events, or objects with certain qualifications within a study, was used for sampling purposes (Büyüköztürk et al., 2008). The first criterion for inclusion in the study was being a teacher candidate who had taken courses in geometry and topology; the second was having taken most of the field and training courses in the program, which suggested that a prospective participant had an informed opinion on topology.

### 2.2 | Data collection tools

One of the data collection tools used in the study consisted of four geometry questions available in the literature.

Question 1 was taken from Hızarcı and Konyalıoğlu (2010); Question 2 was taken from Hızarcı et al. (2015); Question 3 was taken from Kaplan et al. (2011); and Question 4 was prepared from the authors. All of these questions (see the Appendix A), which were included in the data collection tool, can be solved by applying topological thought to plane geometry.

The second data collection tool used was a form (see the Appendix B and C): the participants were asked to provide semi-structured written opinions of their views on undergraduate courses and their reflections on secondary education.

### 2.3 | Process and data analysis

Data for the study were obtained via a paper that the participants were asked to fill out by hand in a classroom environment. Before starting the study, it was explained to the participants that individuals would not be given marks and the topic of the study was mentioned. In the study, the solutions to the four teachers' questions included in the data collection tool were examined and the responses provided were divided into topological and plane geometry categories. The data obtained from the answers given on the second data collection tool were then analyzed using descriptive and content analysis, in which data are analyzed according to the conceptual structure and purpose of the study and the code, categories, and themes determined beforehand (Yıldırım & Şimşek, 2013). In order to ensure reliability during the data analysis process, the codings were carried out separately by three mathematics educators. Then, it was discussed on the points that differed from the viewpoint and consensus was achieved on the codings. The data obtained from the answers provided by the teacher candidates on the second data collection tool were classified into categories (e.g., the most abstract course, the most useful course, the least useful course, and the course that will be the most useful for them when they become teachers) according to the purpose of the study. The data analyzed are presented to the reader in tables and almost all answers presented in each category are supported by direct quotations.

The opinions of the teacher candidates about the undergraduate courses were collected using a structured written opinion form, which aims to reveal the place of topology in the teacher candidates' professional careers.

## 3 | RESULTS

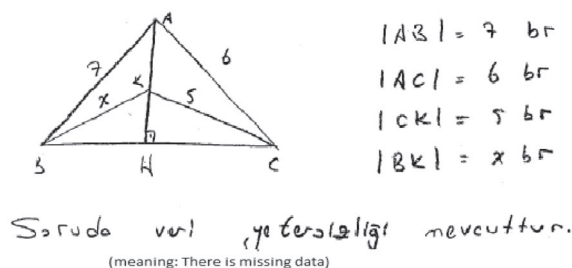
The participants' answers to the first data collection tool were analyzed in accordance with the purposes of the study. The participants' opinions about the undergraduate courses are presented below in the form of tables.

The teacher candidates' approaches to geometry questions and how to solve questions are presented below in Table 1.

The statements included in Table 1 reveal that none of the teacher candidates applied topology or topological thinking to the mathematical problems they were given; instead, they tried to find the solution using the rules of plane geometry. Fifteen candidates for Question 1, 17 candidates for Question 2, and 13 candidates for Question 4 stated that there were either missing data or a lack of data in the questions. In Question 3, two candidates said the question was incomplete and only four were able to solve the problem.

In answering the first question, 7 of the teacher candidates obtained the correct solution using plane geometry; 9 used plane geometry but reached an incorrect solution; 15 stated that the question did not provide sufficient information; and 5 did not attempt to provide an answer. Some of the solutions and answers given for the first question are detailed below:

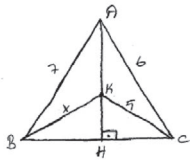
T.8's solution:



**TABLE 1** The data about question solving styles of the teacher candidates

	There is a solution				There is no solution			
	Topological		Traditional		Incomplete question	Incorrect question	Empty	Total
	D	Y	D	Y				
Question 1	0	0	7	9	15	0	5	36
Question 2	0	0	8	1	17	0	10	36
Question 3	0	0	4	26	2	3	1	36
Question 4	0	0	0	6	13	2	15	36

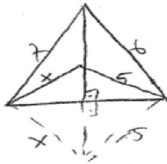
T.15's solution:



$|AB| = 7br$   
 $|AC| = 6br$   
 $|CK| = 5br$   
 $|BK| = xbr$   
 $x = ?$

Soruda eksik var!  
(meaning: lack of data)

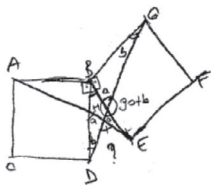
T.27's solution:



$x^2 + 6^2 = 7^2 + 5^2$   
 $x^2 + 36 = 49 + 25$   
 $x^2 = 38$   
 $x = \sqrt{38}$

In answering the second question, none of the teachers used a topological approach. Eight of the teacher candidates obtained the correct solution using plane geometry; 1 used plane geometry but reached the wrong solution; 17 teacher candidates stated that the question did not provide sufficient information; and 10 candidates did not attempt to provide an answer. Some of the teacher candidates' answers to the second question were as follows:

T.5's solution:



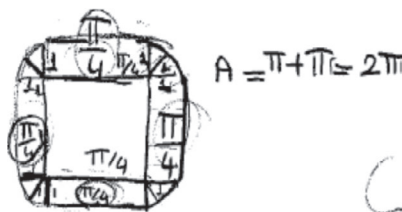
$ABCD$  ve  $EFGH$  için  
 $2\alpha + 180 = 360$   
 $2\alpha = 180$   
 $\alpha = 90$

T.26's response:

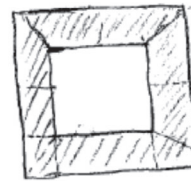
Eksik verili bir soru. (meaning: a question that lacks data)  
Karelerin birbirine eşit oldukları ve B açısı verilmeliydi.

In answering the third question, the participants again declined to use a topological approach. Four candidates answered the question correctly using plane geometry; 26 used plane geometry but obtained the wrong solution; 2 stated that the question did not provide sufficient information; 3 stated that the problem was wrong; and 1 candidate did not respond to the question at all. Some of the answers were as follows:

T.6's solution:



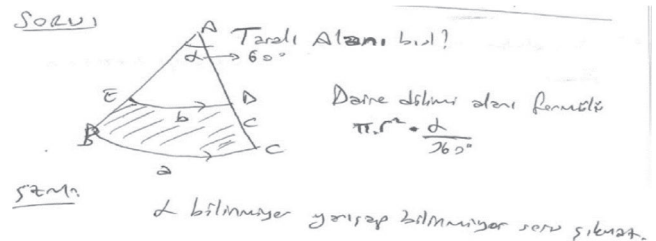
T.34's solution:



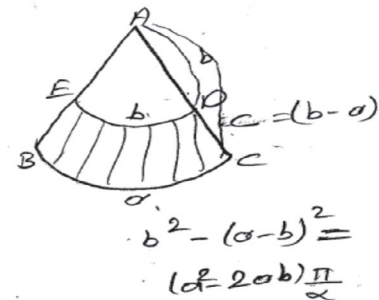
Çevre =  $\pi$  ise  
 bir kenarı  $\frac{\pi}{4}$   
 karenin alanı  $\frac{\pi^2}{16}$

In answering the fourth question, none of the teacher candidates used a topological approach. In this case, interestingly, none of the teacher candidates obtained the correct solution; 6 tried to solve the question using plane geometry rules but got the answer wrong; 13 stated that the question did not provide sufficient information; 2 stated that the problem was wrong; and 15 candidates left the solution partially empty.

T.18's solution:



T.33's solution:



The above findings suggest that teacher candidates' use of topology and topological thinking are not sufficient. While this matter could be discussed in a nonobjective fashion, the qualifications of teacher candidates are a matter of concern.

Teacher candidates' opinions about the undergraduate courses were collected with a written opinion form in order to reveal the place of topology in the teacher candidates' professional careers, which is the actual subject of this study. The written feedback from the 36 teacher candidates relating to a few undergraduate courses were categorized as follows: the most abstract course, the most hands-on course, the most useful course, the least useful course, and the course that will be the most useful for them when they become teachers. These opinions were categorized in an attempt to determine how important the participants felt the topology course was to their education, life, and teaching.



An examination of the explanations presented in Table 2 revealed that the word geometry was not included in any of the candidates' definitions. Seventy-five percent considered topology to be a subject that dealt with the properties of spaces, and when the candidates were asked about geometry, only one considered topology to be a part of geometry. Regarding the coding, some teacher candidates' sample statements are given below.

"Topology is a space math." (TC-5)

"Some operations and concepts in spaces." (TC-17).

"The inner world of spaces and sets." (TC-23).

"Space science." (TC-11).

Table 3 below presents most of the data gathered, including the candidates' opinions about undergraduate courses:

When we look at the categories above, the 36 participants had varying opinions regarding which course was the most abstract: 20 said topology; 10 said abstract algebra; 2 said abstract mathematics; 2 said differential geometry; 1 said linear algebra; and 1 said functional analysis. When

**TABLE 2** The definition of teacher candidates about topology

Definitions	<i>f</i>	%
The discipline that examines the properties of spaces (metric spaces)	27	75
The widest space with no relation to length	1	3
Clusters and groups (group, ring) viewing branch	4	11
A lesson that makes different thinking possible	1	3
Distance	1	3
Scientist examining spaces and clusters	2	6

**TABLE 3** Teacher candidates' thoughts on undergraduate courses

Courses	The most useful course		The least useful course		The most abstract course		The most concrete course	
	<i>f</i>	%	<i>f</i>	%	<i>f</i>	%	<i>f</i>	%
Topology	1	3	3	8	20	56	–	–
Analysis	30	83	–	–	–	–	12	33
Abstract algebra	–	–	–	–	10	28	–	–
Abstract mathematics	–	–	–	–	2	6	–	–
Linear Algebra	1	3	–	–	1	3	5	14
Numbers theory	1	3	–	–	–	–	3	8
Geometry	1	3	–	–	–	–	5	14
Applied mathematics	1	3	2	6	–	–	6	17
Differential geometry	–	–	15	42	2	6	–	–
Functional analysis	–	–	2	6	1	3	–	–
Statistics	–	–	–	–	–	–	3	8
Tensor analysis	–	–	6	17	–	–	–	–

asked which course was the most concrete, the candidates' responded: 12 analysis courses; 6 applied mathematics; 5 geometry; 5 linear algebra; 3 number theory; 3 statistics; and 2 no opinion. When they were asked which courses they thought were the most useful for their university education, they answered as follows: 30 candidates analysis; 1 linear algebra; 1 number theory; 1 geometry; 1 applied mathematics; and 1 topology. When asked which courses they thought were the least useful, they replied as follows: 15 candidates differential geometry; 4 tensor analysis; 3 topology; 2 functional analysis; 2 numerical analysis; 2 applied mathematics; and 8 no comment. Finally, when the candidates were asked which course they expect to be most useful in their teaching career, they answered as follows: 29 analysis courses; 2 applied mathematics; 2 abstract algebra; 1 linear algebra; 1 number theory; 1 geometry; and 1 abstract mathematics.

## 4 | DISCUSSION

In this study, the teacher candidates' geometric thinking abilities and opinions about their undergraduate courses were examined in order to determine what they thought of their topology course. Although children acquire an instinctive grasp of topology before they begin formal education, the teaching process unfortunately eliminates the effectiveness of this system of thought over time (Greenstein, 2014). Topology is perceived as a high-level mathematics subject that most students do encounter until they begin university life. Piaget stated that it is wrong to start teaching geometry using Euclidean geometry, because the first way children perceive space and their environment is topological (Clements, 2004; Piaget &

Inhelder, 1967). According to Piaget, in the first stage of learning, children understand the topological properties of the objects around them through sensory-motor activities and can distinguish shapes according to these characteristics. In this sense, a non-square, rectangle, or even a triangle or a square can be seen as an image that can be converted. Thus, the square, rectangular, and triangle appear to have the same rather than distinct qualities (Greenstein, 2014). According to Piaget, children can only recognize and distinguish between Euclidean forms such as triangles, rectangles, and squares at the second stage of learning (Piaget & Inhelder, 1967). In order for children to understand geometric shapes more accurately and permanently, it is very important to present both topological thinking and Euclidean thinking together (Aslan & Arnas, 2007; Clements, 1999).

In Euclidean geometry, unchanging properties are discussed through the application of reflection, translation, rotation, and transformation; the distance between two points on the shape is always preserved (Arnold, 1963, p. 22). In topological geometry, though, the characteristics of inner-outer and end points, order of the points and connectedness are preserved (Kidder, 1976). Ascertaining the topological structure of a shape requires the use of both geometric intuition and imagination. In other words, a student who is working on topological transformations is expected to imagine what form a shape can be transformed into by bending and twisting it (Delice & Karaaslan, 2016). As a result of such studies, it is expected that spatial thinking (Clements & Battista, 1992), which is also defined as the ability to play on the properties of the objects, develops. When addressed within the scope of geometry, topology preserves a smaller number of features with respect to the structure of a shape, allowing for elastic movements such as displacement, pulling, bending, and the possibility of looking at a shape from a different point of view. Delice and Karaaslan (2016) stated that teaching this concept is important in the field of mathematics, which includes clusters and geometry, because it allows for a shape to be reduced to simple levels; this complies with the approach of the new program in terms of reflecting the developing mathematics field.

In this study, all of the questions directed to the teacher candidates were prepared in such a way that they could be solved using topological thinking rather than simple plane geometry. Topological thinking allows students to obtain a solution by transforming the shapes they were familiar with from Euclidean geometry into new forms. However, it was found that none of the teacher candidates used topological thinking to solve the questions. In order to determine why this happened, it was necessary to assess how much importance the participants placed on topology in their educational lives and undergraduate education. In their study, Delice and Karaaslan (2016) stated that topology refers to basic concepts that can be felt more intuitively (e.g., interior, exterior,

open curve, closed curve, sequence, and genus) in an informal way in at the elementary, middle, and high school levels. However, it has been noted that while topology is taught at the undergraduate level in other countries, in Turkey topology is studied as a part of the field of mathematics rather than mathematics education. The undergraduate program of mathematics department consists of eight semesters. In the program, there are courses which sum up to 240 ECTS credits that split up to 30 ECTS credits per semester. In addition, in the program teacher candidates have to take two (2) topology courses in total (12 ECTS, 5%). However, it might not be referenced in such a way that teacher candidates should be able to draw upon the knowledge (as opposed to plane geometry). On the contrary, it is predicted that more than half of teacher candidates who see topology as abstract may also have an influence on these views of the candidates. When the answers given to define topology were examined, it was found that none of the candidates had used the word geometry.

Consequently, teacher candidates do not use topological thinking to solve mathematical problems because topology has been neglected in their primary and secondary education. We predicted that if the participants had encountered topology more often prior to beginning college or university, they would have had the chance to put forth different perspectives and more creative solutions to the questions they were asked.

Including topology in primary school classes will not only provide an enriching experience for students, but will also create an opportunity to evaluate the effectiveness of many reformist ideas regarding mathematics education as a whole (Sugarman, 2014). In his research, Sugarman (2014) studied the effect of teaching topology to students in grades 4 to 6 for two lessons. At the end of the study, it was found that there were significant increases in the numbers of students who questioned mathematics and those who saw mathematics as a creative subject. Introducing topology to young students is a strong example of how student perceptions of math can be broadened to set the stage for increased participation in, and excitement about, the field of mathematics (Sugarman, 2014).

## 5 | CONCLUSION AND RECOMMENDATIONS

The most striking finding of this study is that the participants did not feel that topology and topological thought are particularly significant to human life. One of the main reasons for this is that in formal training topological thought not given as much attention as it deserved. It is less than ideal to suspect that a majority of future math teachers may lack a sufficient understanding of topology, which they may one day be called upon to teach. Those candidates who considered topology to be an abstract course were unable

to see its relationship to with geometry. Candidates trained in Euclidean geometry throughout their primary and secondary education still identified topology as being very different from geometry, despite having been taught different geometric systems at university.

Future teachers must be taught topology and the topological thought, and to do so it is necessary to give this topic the place it deserves in the educational process. Introducing topology into the educational process early on will enrich the perspectives of young students on mathematics. Paying attention to the recommendations of Delice and Karaaslan (2016) and Kaplan et al. (2011), and considering the limited studies available on the teaching of topology, it is necessary to research whether the subject should be included at the undergraduate level in Turkey, what concepts should be included, and why.

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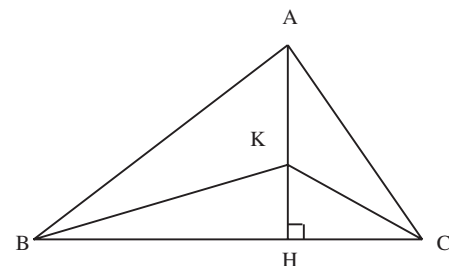
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## APPENDIX A

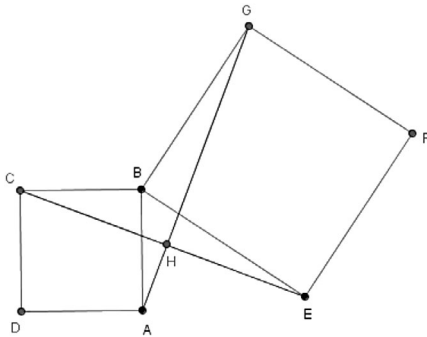
### Geometry questions

#### Question 1



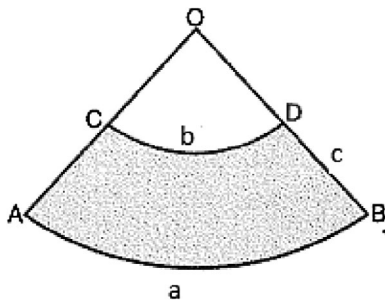


Question 2. In the figure below, ABCD and EFGH are two squares. Find the angle of (AHE).



Question 3. What is the area of the region of points which remain outside a triangle, the perimeter of which is  $\pi$  and whose distance to the triangle does not exceed 1?

Question 4. What is the area of the shaded zone a,b,c?



## APPENDIX B

### Teacher candidates' definitions for each course

Courses	Your definition
Topology:	
Analysis:	
Abstract algebra:	
Abstract mathematics:	
Linear algebra:	
Numbers theory:	
Geometry:	
Applied mathematics:	
Differential geometry:	
Functional analysis:	
Statistics:	
Tensor analysis:	

## APPENDIX C

### Teacher candidates' thoughts on undergraduate courses

Courses	Reason
The most useful courses:	
The least useful courses:	
The most abstract courses:	
The most concrete courses:	
The most hands-on courses:	
The most useful courses in your future teaching career:	
The courses that include geometry concepts:	