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Numerical solutions of the modified KdV Equation with collocation method

Seydi Battal Gazi Karakoc[1]*

Abstract

In this article, numerical solutions of the modified Korteweg-de Vries (MKdV) equation have been obtained by a numerical technique attributed on collocation method using quintic B-spline finite elements. The suggested numerical scheme is controlled by applying three test problems involving single solitary wave, interaction of two and three solitary waves. To check the performance of the newly applied method, the error norms, L_2 and L_∞, as well as the three lowest invariants, I_1, I_2 and I_3, have been calculated. The acquired numerical results are compared with some of those available in the literature. Linear stability analysis of the algorithm is also examined.

Keywords

Modified Korteweg-de Vries equation; finite element method; collocation; quintic B-spline; soliton.

AMS Subject Classification

65N30, 65D07, 74S05,74J35, 76B25.

[1] *Department of Mathematics, Faculty of Science and Art, Nevsehir Haci Bektas Veli University, Nevsehir, 50300, Turkey.*

***Corresponding author**: [1] sbgkarakoc@nevsehir.edu.tr

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Contents

1. Introduction

This article is concerned with the following non-linear modified Korteweg de-Vries (MKdV) equation

$$U_t + \varepsilon U^2 U_x + \mu U_{xxx} = 0, \tag{1.1}$$

with the homogeneous boundary conditions

$$\begin{aligned} U(a,t) &= 0, \qquad U(b,t) = 0, \\ U_x(a,t) &= 0, \qquad U_x(b,t) = 0, \quad t > 0 \end{aligned} \tag{1.2}$$

and an initial condition

$$U(x,0) = f(x) \quad a \le x \le b \tag{1.3}$$

where t is time, x is the space coordinate, ε and μ are positive parameters and $f(x)$ is a detected function. A main mathematical model for describing the theory of water waves in shallow channels is the following Korteweg de-Vries (KdV) equation:

$$U_t + \varepsilon U U_x + \mu U_{xxx} = 0. \tag{1.4}$$

The terms UU_x and U_{xxx} in the Eq.(1.4) represent the nonlinear convection and dispersion, respectively. Many physical phenomena for example propagation of long waves in shallow water waves, bubble-liquid mixtures, ion acoustic plasma waves and wave phenomena in enharmonic crystals can be described by the KdV equation which was first introduced by Korteweg and de Vries [1]. The exact solutions of the equation obtained by [2, 3]. KdV equation was first solved numerically by Zabusky and Kruskal using finite difference method [4]. Gardner et al. [5] showed the existence and uniqueness of solutions of the KdV equation. Many researches have used various numerical methods including finite difference method [6, 7], finite element method [8, 15], pseudospectral method [3] and heat balance integral method [16] to solve the equation. MKdV equation have a limited number of numerical studies in the literature. Kaya [17], was used the Adomian decomposition method to obtain the higher order modified Korteweg de-Vries equation with initial condition. MKdV equation have been solve by using Galerkins' method with quadratic B-spline finite elements by Biswas et al. [18]. Raslan and Baghdady [19, 20], showed the accuracy and stability of the difference solution of the MKdV equation and they obtained the numerical aspects of the dynamics of shallow water waves along lakes' shores and beaches modeled by the MKdV equation. A new variety of (3 + 1)-dimensional modified Korteweg–de Vries (mKdV) equations and multiple soliton solutions for each new equation were established by Wazwaz [21, 22]. A lumped Galerkin and Petrov Galerkin methods were applied to the MKdV equation by Ak et al. [23, 24].

In this paper, we have numerically solve the MKdV equation using collocation method with quintic B-spline finite elements. We have studied the motion of a single solitary wave, interaction of two and three solitary waves to show the performance and efficiency of the suggested method. We showed the proposed method is unconditionally stable applying the von-Neumann stability analysis.

2. Quintic B-spline Collocation Method

For our numerical computations, solution area of the problem is limited over an interval $a \le x \le b$. Let the partition of the space interval $[a,b]$ into equally sized finite elements of length h at the points x_m like that $a = x_0 < x_1 < ... < x_N = b$ and $h = \frac{b-a}{N}$. The set of quintic B-spline functions $\{\phi_{-2}(x), \phi_{-1}(x), \dots, \phi_{N+1}(x), \phi_{N+2}(x)\}$ form a basis over the solution region $[a,b]$. The numerical solution $U_N(x,t)$ is expressed in terms of the quintic B-splines as

$$U_N(x,t) = \sum_{m=-2}^{N+2} \phi_m(x)\delta_m(t) \tag{2.1}$$

where $\delta_m(t)$ are time dependent parameters and will be defined from the boundary and collocation conditions. Quintic B-splines $\phi_m(x)$, $(m = -2,-1,...,N+1,N+2)$ at the knots x_m are designated over the interval $[a,b]$ by Prenter [25]

$$\phi_m(x) = \frac{1}{h^5}\begin{cases} a^5, & [x_{m-3}, x_{m-2}] \\ a^5 - 6b^5, & [x_{m-2}, x_{m-1}] \\ a^5 - 6b^5 + 15c^5, & [x_{m-1}, x_m] \\ a^5 - 6b^5 + 15c^5 - 20d^5, & [x_m, x_{m+1}] \\ a^5 - 6b^5 + 15c^5 - 20d^5 & \\ +15e^5, & [x_{m+1}, x_{m+2}] \\ a^5 - 6b^5 + 15c^5 - 20d^5 & \\ +15e^5 - 6f^5, & [x_{m+2}, x_{m+3}] \\ 0, & elsewhere \end{cases} \tag{2.2}$$

where $a = (x - x_{m-3})$, $b = (x - x_{m-2})$, $c = (x - x_{m-1})$, $d = (x - x_m)$, $e = (x - x_{m+1})$, $f = (x - x_{m+2})$. Each quintic B-spline covers six elements, thus each element $[x_m, x_{m+1}]$ is covered by six splines. A typical finite interval $[x_m, x_{m+1}]$ is mapped to the interval $[0,1]$ by a local coordinate transformation defined by $h\xi = x - x_m$, $0 \le \xi \le 1$. So quintic B-splines (2.2) in terms of ξ over $[0,1]$ can be given as follows:

$$\begin{aligned} \phi_{m-2} &= 1 - 5\xi + 10\xi^2 - 10\xi^3 + 5\xi^4 - \xi^5, \\ \phi_{m-1} &= 26 - 50\xi + 20\xi^2 + 20\xi^3 - 204\xi^4 + 5\xi^5, \\ \phi_m &= 66 - 60\xi^2 + 30\xi^4 - 10\xi^5, \\ \phi_{m+1} &= 26 + 50\xi + 20\xi^2 - 20\xi^3 - 20\xi^4 + 5\xi^5, \\ \phi_{m+2} &= 1 + 5\xi + 10\xi^2 + 10\xi^3 + 5\xi^4 - 5\xi^5, \\ \phi_{m+3} &= \xi^5. \end{aligned} \tag{2.3}$$

For the problem, the finite elements are identified with the interval $[x_m, x_{m+1}]$. Using Eq.(2.2) and Eq.(2.1), the nodal values of U_m, U'_m, U''_m, U'''_m and U^{iv}_m are given in terms of the element parameters δ_m by

$$\begin{aligned} &U_N(x_m,t) = U_m = \delta_{m-2} + 26\delta_{m-1} + 66\delta_m + \\ &26\delta_{m+1} + \delta_{m+2}, \\ &U'_m = \tfrac{5}{h}(-\delta_{m-2} - 10\delta_{m-1} + 10\delta_{m+1} + \delta_{m+2}), \\ &U''_m = \tfrac{20}{h^2}(\delta_{m-2} + 2\delta_{m-1} - 6\delta_m + 2\delta_{m+1} + \delta_{m+2}), \\ &U'''_m = \tfrac{60}{h^3}(-\delta_{m-2} + 2\delta_{m-1} - 2\delta_{m+1} + \delta_{m+2}), \\ &U^{iv}_m = \tfrac{120}{h^4}(\delta_{m-2} - 4\delta_{m-1} + 6\delta_m - 4\delta_{m+1} + \delta_{m+2}) \end{aligned} \tag{2.4}$$

and the variation of U over the element $[x_m, x_{m+1}]$ is given by

$$U = \sum_{m=-2}^{N+2} \phi_m \delta_m. \tag{2.5}$$

When we define the collocation points with the knots and use Eqs.(2.4) to utilise U_m, its space derivatives and substitute into Eq. (1.1), this brings to a set of ordinary differential equations of the form

$$\begin{aligned} &\left(\dot{\delta}_{m-2} + 26\dot{\delta}_{m-1} + 66\dot{\delta}_m + 26\dot{\delta}_{m+1} + \dot{\delta}_{m+2}\right) + \\ &\varepsilon \tfrac{5}{h} Z_m(-\delta_{m-2} - 10\delta_{m-1} + 10\delta_{m+1} + \delta_{m+2}) + \\ &\mu \tfrac{60}{h^3}(-\delta_{m-2} + 2\delta_{m-1} - 2\delta_{m+1} + \delta_{m+2}) = 0, \end{aligned} \tag{2.6}$$

where

$$Z_m = (\delta_{m-2} + 26\delta_{m-1} + 66\delta_m + 26\delta_{m+1} + \delta_{m+2})^p.$$

If time parameters δ_i and its time derivatives $\dot{\delta}_i$ in Eq.(2.6) are discretized by the Crank-Nicolson formula

$$\delta_i = \frac{\delta_i^{n+1} + \delta_i^n}{2}, \tag{2.7}$$

and usual finite difference aproximation

$$\dot{\delta}_i = \frac{\delta_i^{n+1} - \delta_i^n}{\Delta t} \tag{2.8}$$

we derive a repetition relationship between two time levels n and $n+1$ relating two unknown parameters δ_i^{n+1}, δ_i^n for $i = m-2, ..., m+2$

$$
\begin{aligned}
& \gamma_1 \delta_{m-2}^{n+1} + \gamma_2 \delta_{m-1}^{n+1} + \gamma_3 \delta_m^{n+1} + \gamma_4 \delta_{m+1}^{n+1} + \gamma_5 \delta_{m+2}^{n+1} \\
= \; & \gamma_5 \delta_{m-2}^{n} + \gamma_4 \delta_{m-1}^{n} + \gamma_3 \delta_m^{n} + \gamma_2 \delta_{m+1}^{n} + \gamma_1 \delta_{m+2}^{n}
\end{aligned} \tag{2.9}
$$

where

$$
\begin{aligned}
&\gamma_1 = [1 - EZ_m - M], \\
&\gamma_2 = [26 - 10EZ_m + 2M], \\
&\gamma_3 = [66], \\
&\gamma_4 = [26 + 10EZ_m - 2M], \\
&\gamma_5 = [1 + EZ_m + M], \\
&m = 0, 1, \ldots, N, E = \tfrac{5}{2h}\varepsilon\Delta t, M = \tfrac{30}{h^3}\mu\Delta t.
\end{aligned} \tag{2.10}
$$

The system (2.9) involves of $(N+1)$ linear equations containing $(N+5)$ unknown coefficients $(\delta_{-2}, \delta_{-1}, \ldots, \delta_{N+1}, \delta_{N+2})^T$. We need four additional restraints to obtain a unique solution for this system. These are obtained from the boundary conditions (1.2) and can be used to remove δ_{-2}, δ_{-1} and $\delta_{N+1}, \delta_{N+2}$, from the systems (2.9) which occures a matrix equation for the $N+1$ unknowns $d^n = (\delta_0, \delta_1, \ldots, \delta_N)^T$ of the form

$$
A\mathbf{d^{n+1}} = B\mathbf{d^{n}}. \tag{2.11}
$$

The matrices A and B are $(N+1) \times (N+1)$ penta-diagonal matrices and this matrix equation have been solved by using the penta-diagonal algorithm. However, two or three inner iterations are implemented to the term $\delta^{n*} = \delta^n + \frac{1}{2}(\delta^n - \delta^{n-1})$ at each time step to overcome the non-linearity caused by Z_m. Before the solution process begins iteratively, the initial vector d^0 must be established by using the initial condition and following derivatives at the boundaries;

$$
\begin{aligned}
U_N(x,0) &= U(x_m,0); \qquad m = 0,1,2,...,N \\
(U_N)_x(a,0) &= 0, \qquad (U_N)_x(b,0) = 0.
\end{aligned}
$$

So we have the following matrix form for the initial vector d^0;

$$
Vd^0 = w,
$$

where

$$
V = \begin{bmatrix}
54 & 60 & 6 & & & & & & \\
25.25 & 67.50 & 26.25 & 1 & & & & & \\
1 & 26 & 66 & 26 & 1 & & & & \\
 & 1 & 26 & 66 & 26 & 1 & & & \\
 & & & & \ddots & & & & \\
 & & & 1 & 26 & 66 & 26 & 1 \\
 & & & & 1 & 26.25 & 67.50 & 25.25 \\
 & & & & & 6 & 60 & 54
\end{bmatrix}
$$

$d^0 = (\delta_0, \delta_1, \delta_2, ..., \delta_{N-2}, \delta_{N-1}, \delta_N)^T$ and
$w = (U(x_0,0), U(x_1,0), ..., U(x_{N-1},0), U(x_N,0))^T$.

3. Stability Analysis

In order to examine the stability analysis of the suggested scheme, it is properly to use Von-Neumann theory. Presuming that the quantity U^2 in the nonlinear term U^2U_x is locally constant. Substituting the Fourier mode $\delta_m^n = \xi^n e^{i\sigma mh}, (i = \sqrt{-1})$ into the form of (2.9) we attain,

$$
\begin{aligned}
& \xi^{n+1}(\eta_1 e^{i(m-2)\theta} + \eta_2 e^{i(m-1)\theta} + \eta_3 e^{im\theta} + \\
& \eta_4 e^{i(m+1)\theta} + \eta_5 e^{i(m+2)\theta}) \\
= \; & \xi^n(\eta_5 e^{i(m-2)\theta} + \eta_4 e^{i(m-1)\theta} + \\
& \eta_3 e^{im\theta} + \eta_2 e^{i(m+1)\theta} + \\
& \eta_1 e^{i(m+2)\theta})
\end{aligned} \tag{3.1}
$$

where σ is mode number, h is the element size, $\theta = \sigma h$

$$
\begin{aligned}
&\eta_1 = 1 - \beta_1 - \beta_2, \\
&\eta_2 = 26 - 10\beta_1 + 2\beta_2, \\
&\eta_3 = 66, \\
&\eta_4 = 26 + 10\beta_1 - 2\beta_2, \\
&\eta_5 = 1 + \beta_1 + \beta_2, \\
&m = 0, 1, \ldots, N, \quad \beta_1 = \tfrac{5}{2h}\varepsilon\Delta t, \; \beta_2 = \tfrac{30}{h^3}\mu\Delta t.
\end{aligned}
$$

If we simplify the Eq. (3.1),

$$
\xi = \frac{A + iB}{A - iB}
$$

is obtained where

$$
\begin{aligned}
A &= (52)\cos(\theta) + (2)\cos(2\theta) + 66 \\
B &= 4(5EZ_m - M)\sin(\theta) + 2(EZ_m + M)\sin(2\theta).
\end{aligned} \tag{3.2}
$$

According to the Fourier stability analysis, for the given scheme to be stable, the condition $|\xi| < 1$ must be satisfied. Using a symbolic programming software or using simple calculations, since $a^2 + b^2 = a^2 + (-b)^2$ it becomes evident that the modulus of $|\xi|$ is 1. Therefore the linearized scheme is unconditionally stable.

4. Numerical Results and Discussion

In this part, to confirm the correction of our scheme, some numerical experiments were calculated: the motion of single solitary wave whose analytical solution is known and extended the scheme to the study of two and three solitary waves, whose analytical solution is unknown during the interaction. The initial boundary value problem (1.1) – (1.3) possesses following conservative quantities;

$$
\begin{aligned}
I_1 &= \textstyle\int_{-\infty}^{\infty} U(x,t)dx, \\
I_2 &= \textstyle\int_{-\infty}^{\infty} U^2(x,t)dx, \\
I_3 &= \textstyle\int_{-\infty}^{\infty} [U^4(x,t) - \frac{6\mu}{\varepsilon} U_x^2(x,t)]dx
\end{aligned} \tag{4.1}
$$

which correspond to the mass, momentum and energy of the shallow water waves, respectively[26**?**]. To calculate the difference between analytical and numerical solutions at some specified times, the error norm L_2

$$
L_2 = \left\|U^{exact} - U_N\right\|_2 \simeq \sqrt{h \sum_{J=0}^{N} \left|U_j^{exact} - (U_N)_j\right|^2},
$$

and the error norm L_∞

$$
L_\infty = \left\|U^{exact} - U_N\right\|_\infty \simeq \max_j \left|U_j^{exact} - (U_N)_j\right|
$$

have been used.

4.1 The motion of single solitary wave

For this problem, Eq.(1.1) is analyzed with the boundary conditions $U \to 0$ as $x \to \pm\infty$ and the initial condition

$$U(x,0) = A \sec h[k(x-x_0)] \tag{4.2}$$

where $A = \sqrt{\frac{6c}{\varepsilon}}$, $k = \sqrt{\frac{c}{\mu}}$ and A is amplitude, k is the width of the single solitary wave. The exact solution of the MKdV equation can be written as

$$U(x,t) = A \sec h[k(x-ct-x_0)] \tag{4.3}$$

where ε, μ, c, and x_0 are arbitrary constants. For this problem, the analytical values of the invariants can be given as [18]

$$I_1 = \pi\sqrt{\frac{6\mu}{\varepsilon}}, I_2 = \frac{12\sqrt{\mu c}}{\varepsilon}, I_3 = -\frac{64c^2}{\varepsilon^2}\sqrt{\frac{\mu}{c}}. \tag{4.4}$$

For the computational study, we have chosen the parameters $\varepsilon = 3$, $\mu = 1$, $h = 0.1$, $c = 0.845$ and $\Delta t = 0.01$ through the interval $0 \le x \le 80$, so the solitary wave has amplitude $A = 1.3$. The numerical simulations are run to time $t = 20$ to find error norms L_2, L_∞ and conserved quantities I_1, I_2 and I_3. Comparisons of the values of the invariants and error norms provided by the suggested method with those obtained some earlier methods are given in Table (1). From this table, it is obviously seen that the error norms obtained by our method are found much better than the others and the computed values of invariants are in good agreement with their analytical values. Solitary wave profiles are demonstrated at different time levels in Fig.(1) in which the soliton moves to the right at a nearly unchanged speed and amplitude as time increases, as expected.

Table 1. A Comparison of invariants and error norms for single solitary wave with $\varepsilon = 3$, $\mu = 1$, $c = 0.845$, $h = 0.1$ and $\Delta t = 0.01$, $0 \le x \le 80$.

t		1	10	20
I_1	*Present*	4.44286	4.44282	4.44278
	[18]	4.44300	4.44414	4.44317
	[23]	4.44286	4.44286	4.44286
	[24]	4.44286	4.44286	4.44286
I_2	*Present*	3.67693	3.67686	3.67678
	[18]	3.67706	3.67809	3.67919
	[23]	3.67694	3.67694	3.67694
	[24]	3.67694	3.67694	3.67694
I_3	*Present*	2.07131	2.07119	2.07106
	[18]	2.07357	2.07530	2.07716
	[23]	2.07279	2.07369	2.07384
	[24]	2.07279	2.07369	2.07384
L_2	*Present*	2.03E-04	3.24E-04	3.98E-04
	[18]	-	-	-
	[23]	6.27E-04	2.13E-03	3.65E-03
	[24]	6.28E-04	2.13E-03	3.64E-03
L_∞	*Present*	1.43E-05	2.00E-04	6.79E-04
	[18]	1.20E-03	5.94E-03	8.64E-03
	[23]	3.62E-04	1.40E-03	2.29E-03
	[24]	3.63E-04	1.39E-03	2.28E-03

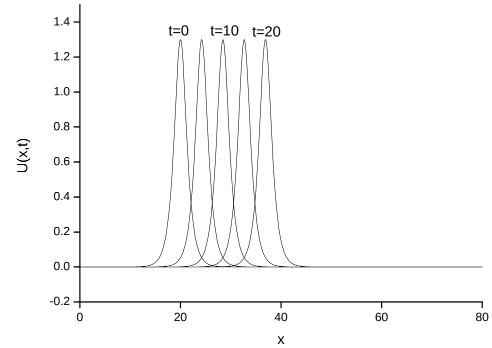


Figure 1. Single solitary wave with $\varepsilon = 3$, $\mu = 1$, $c = 0.845$, $h = 0.1$, $\Delta t = 0.01$ and $0 \le x \le 80$ at $t = 0, 5, 10, 15,$and 20.

4.2 Interaction of two solitary waves

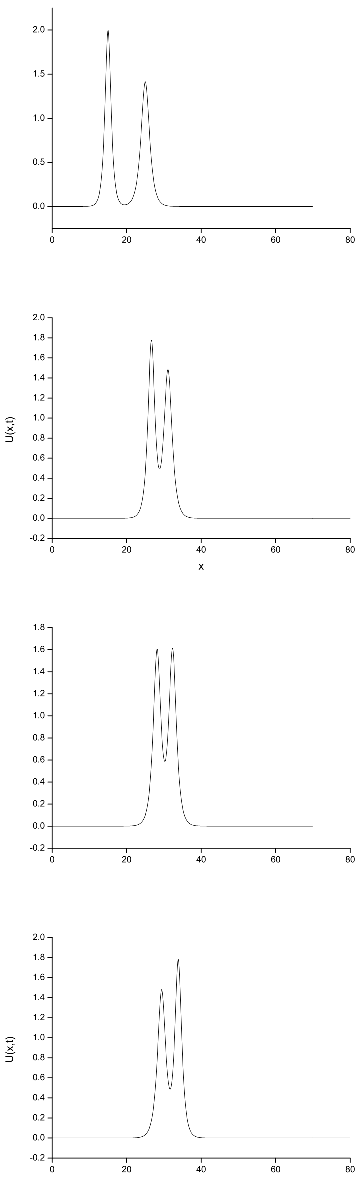
As a second problem, we have discussed the behavior of the interaction of two solitary waves having different amplitudes and travelling in the same direction. Initial condition of two well-seperated solitary waves of different amplitudes has the following form:

$$U(x,0) = \sum_{j=1}^{2} A_j \sec h\left[c_j\left(x-x_j\right)\right] \tag{4.5}$$

$(j = 1,2)$ c_j and x_j are arbitrary constants. To ensure an interaction of two solitary waves we have taken the parameters $\varepsilon = 3$, $\mu = 1$, $h = 0.1$, $\Delta t = 0.01$, $c_1 = 2$, $c_2 = 1$, $x_1 = 15$ and $x_2 = 25$ over the interval $0 \le x \le 80$ to congruent with those used by earlier studies [18, 23, 24]. The run of the algorithm is carried up to time $t = 20$ to obtain the values of the invariants. The obtained results are tabulated in Table (2). Table (2) shows that invariants are nearly constant as the time progresses. Therefore, we can say our method is marginally conservative. The interaction of two solitary waves is depicted at different time levels in Figure (2). It is understood from this figure that at $t = 0$ the wave with larger amplitude which has 2.0 amplitude, is located at the left of the smaller soliton which has 1.414216 amplitude initially. Since the taller wave moves faster than the shorter one, it catches up and collides with the shorter one at $t = 6$ and then moves away from the shorter one as time increases. When the interaction finishes at time $t = 16$, two solitons preserve their originally characteristics like the beginning location. At $t = 20$, the amplitude of larger wave is 2.0 at the point $x = 57.5$ whereas the amplitude of the smaller one is 1.413808 at the point $x = 41.5$. It is found that the absolute difference in amplitude is 4.08×10^{-4} for the smaller wave and 0.0 for the larger wave for this algorithm.

Table 2. A Comparison of invariants for the interaction of two solitary waves with $\varepsilon = 3$, $\mu = 1$, $h = 0.1$, $\Delta t = 0.01$, $c_1 = 2$, $c_2 = 1$, $x_1 = 15$ and $x_2 = 25$, $0 \le x \le 80$.

	t	1	10	20
I_1	*Present*	8.88575	8.88585	8.88577
	[18]	8.88601	8.88974	8.88488
	[23]	8.88573	8.88573	8.88573
	[24]	8.88573	8.88573	8.88573
I_2	*Present*	9.65934	9.65934	9.65934
	[18]	9.65952	9.66254	9.66122
	[23]	9.65934	9.65934	9.65934
	[24]	9.65934	9.65934	9.65934
I_3	*Present*	10.21921	10.21915	10.21920
	[18]	10.23987	10.24679	10.24203
	[23]	10.27082	10.95427	10.33832
	[24]	10.27090	10.95439	10.33841



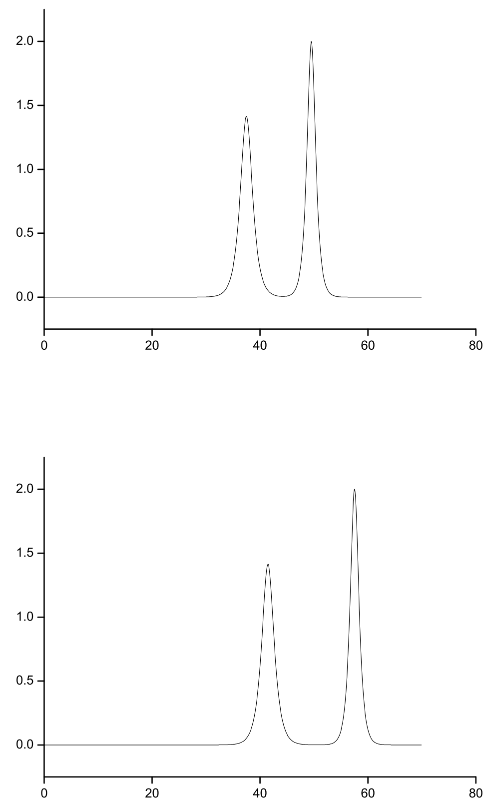


Figure 2. Interaction of two solitary waves with $\varepsilon = 3$, $\mu = 1$, $h = 0.1$, $\Delta t = 0.01$, $c_1 = 2$, $c_2 = 1$, $x_1 = 15$, $x_2 = 25$ and $0 \le x \le 80$ at $t = 0, 6, 7, 8, 16$ and 20.

4.3 Interaction of three solitary waves

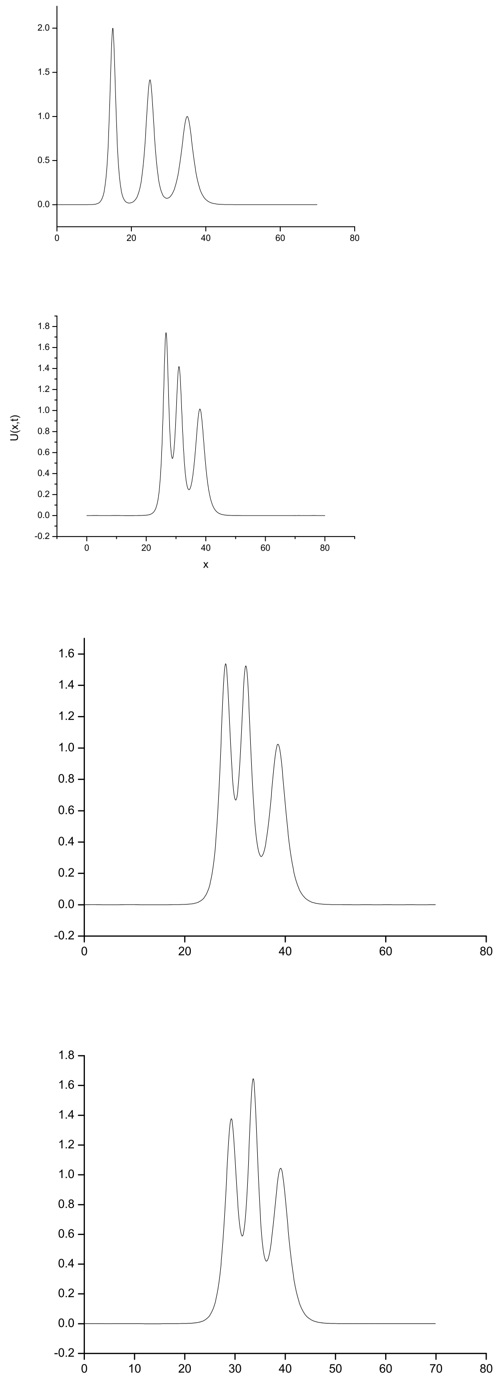
As a final problem, we have considered the behavior of the interaction of three solitary waves having different amplitudes and traveling in the same direction. For our purpose, interaction of three solitary waves is examined by using the initial condition

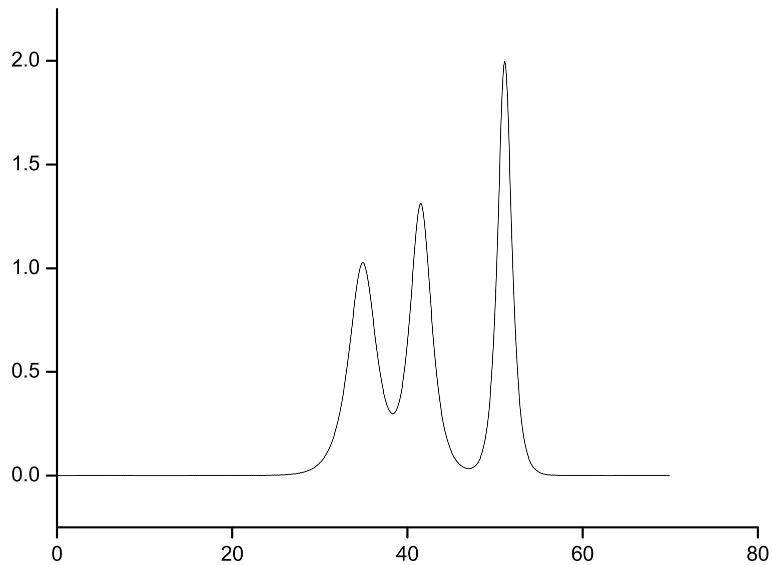
$$U(x,0) = \sum_{j=1}^{3} A_j \operatorname{sec}h\left[c_j\left(x - x_j\right)\right] \tag{4.6}$$

together with boundary conditions $U \to 0$ as $x \to \pm\infty$. This initial condition indicates three solitary waves, one with amplitude A_1 placed initially at $x = x_1$, second with amplitude A_2 placed initially at $x = x_2$ and the last one with amplitude A_3 placed initially at $x = x_3$. We have considered the problem with parameters $\varepsilon = 3$, $\mu = 1$, $h = 0.1$, $\Delta t = 0.01$, $c_1 = 2$, $c_2 = 1$, $c_3 = 0.5$, $x_1 = 15$, $x_2 = 25$ and $x_3 = 35$ over the interval $0 \le x \le 80$ to congruent with those used by earlier studies [18, 23, 24]. The experiment is run from $t = 0$ to $t = 20$ and values of the invariant quantities are listed in Table (3). Table (3) indicates that invariants are nearly constant as the time increases. As one can also see straightforwardly from the table that the values of the invariants are in good agreement with References [18, 23, 24]. The behavior of the interaction of three solitary waves denote at different times in Figure (3).

Table 3. A Comparison of invariants for the interaction of three solitary waves with $\varepsilon = 3$, $\mu = 1$, $h = 0.1$, $\Delta t = 0.01$, $c_1 = 2$, $c_2 = 1$, $c_3 = 0.5$, $x_1 = 15$, $x_2 = 25$ and $x_3 = 35$, $0 \le x \le 80$.

	t	1	10	20
I_1	*Present*	13.32868	13.32915	13.32883
	[18]	13.32906	13.33878	13.33206
	[23]	13.32867	13.32867	13.32867
	[24]	13.32867	13.32867	13.32867
I_2	*Present*	12.51994	12.51995	12.51993
	[18]	12.52028	12.54086	12.52490
	[23]	12.51994	12.51994	12.51994
	[24]	12.51994	12.51994	12.51994
I_3	*Present*	11.22843	11.22802	11.22842
	[18]	11.24979	11.28804	11.25673
	[23]	11.32117	12.41534	11.49914
	[24]	11.32126	12.47629	11.49923





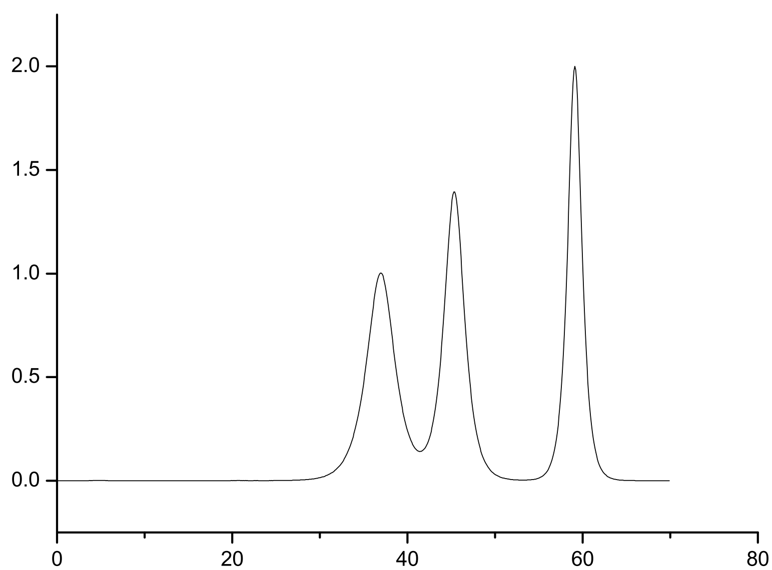


Figure 3. Interaction of three solitary waves with $\varepsilon = 3$, $\mu = 1$, $h = 0.1$, $\Delta t = 0.01$, $c_1 = 2$, $c_2 = 1$, $c_3 = 0.5$, $x_1 = 15$, $x_2 = 25$, $x_3 = 35$ and $0 \leq x \leq 80$ at $t = 0, 6, 7, 8, 16$ and 20.

5. Conclusion

In this paper, we have successfully carried out a quintic B-spline collocation method to the MKdV equation. Three different test problems have been solved. To demonstrate the efficiency of numerical scheme, the error norms L_2, L_∞ and conserved quantities I_1, I_2 and I_3 have been calculated for the test problems. According to the tables in the paper, one can has easily seen that our error norms are enough small and they are better than References [18, 23, 24]. Also, the obtained invariants are acceptable in good agreement with the earlier works [18, 23, 24]. Also, our numerical algorithm is unconditionally stable. So, we can say our numerical algorithm is a reliable method for getting the numerical solutions of the physically important non-linear partial differential equations.

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