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Different Linearization Techniques for the Numerical Solution of the MEW Equation

S. Battal Gazi Karakoc[1], Yusuf Ucar[2], N. Murat Yagmurlu[3]

[1]Department of Mathematics, Faculty of Science and Art, Nevsehir University, Nevsehir, 50300, Turkiye
e-mail: sbgkarakoc@nevsehir.edu.tr
[2]Department of Mathematics, Faculty of Science and Art, Inonu University, Malatya, 44280, Turkiye
e-mail: yusuf.ucar@inonu.edu.tr
[3]Department of Mathematics, Faculty of Science and Art, Inonu University, Malatya, 44280, Turkiye
e-mail: murat.yagmurlu@inonu.edu.tr

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Abstract. The modified equal width wave (MEW) equation is solved numerically by giving two different linearization techniques based on collocation finite element method in which cubic B-splines are used as approximate functions. To support our work three test problems; namely, the motion of a single solitary wave, interaction of two solitary waves and the birth of solitons are studied. Results are compared with other published numerical solutions available in the literature. Accuracy of the proposed method is discussed by computing the numerical conserved laws L_2 and L_∞ error norms. A linear stability analysis of the approximation obtained by the scheme shows that the method is unconditionally stable.

Key words: Finite element method; Collocation; MEW equation; B-Spline; Solitary waves.
2000 Mathematics Subject Classification: 97N40, 65N30, 65D07, 76B25, 74S05, 74J35.

1. Introduction

This paper is concerned with applying the cubic B-spline function to develop a numerical method for approximating the analytic solution of the MEW equation which was introduced by Morrison et al.[9] as a model for nonlinear dispersive waves. This equation has been solved analytically for a limited set of boundary and initial conditions. So the numerical solutions of the MEW equation have been the subject of many studies [1-7,11-19]. In this paper, we have used two different linearization techniques to obtain the numerical solution of the MEW equation. The performance of the method has been tested on three numerical wave propagation experiments: the motion of a single solitary wave, the interaction of two solitary waves and birth of solitons. The stability analysis of the the approximation obtained by the method is also investigated.

2. The Governing Equation and Collocation Solutions

MEW equation takes the form of

$$U_t + 3U^2U_x - \mu U_{xxt} = 0, \qquad a \leq x \leq b \tag{1}$$

with the physical boundary conditions $U \to 0$ as $x \to \pm\infty$, where t is time , x is the space coordinate and μ is a positive parameter. Appropriate boundary conditions will be chosen as

$$\begin{array}{ll} U(a,t) = 0, & U(b,t) = 0, \\ U_x(a,t) = 0, & U_x(b,t) = 0. \end{array} \tag{2}$$

Let us consider the interval $[a,b]$ is partitioned into N finite elements of uniformly equal length by the knots x_i, $i = 0,1,2,...,N$ such that $a = x_0 < x_1 \cdots < x_N = b$ and $h = (x_{i+1} - x_i)$. The cubic B-splines $\phi_i(x)$, (i= -1(1) N+1), at the knots x_i are defined over the interval $[a,b]$ by [8]

(3)

$$\phi_i(x) = \frac{1}{h^3} \begin{cases} (x - x_{i-2})^3, & x \in [x_{i-2}, x_{i-1}], \\ h^3 + 3h^2(x - x_{i-1}) + 3h(x - x_{i-1})^2 - 3(x - x_{i-1})^3, & x \in [x_{i-1}, x_i], \\ h^3 + 3h^2(x_{i+1} - x) + 3h(x_{i+1} - x)^2 - 3(x_{i+1} - x)^3, & x \in [x_i, x_{i+1}], \\ (x_{i+2} - x)^3, & x \in [x_{i+1}, x_{i+2}], \\ 0 & otherwise. \end{cases}$$

The set of splines $\{\phi_{-1}(x), \phi_0(x), \ldots, \phi_{N+1}(x)\}$ forms a basis for the functions defined over [a,b]. Therefore, an approximation solution $U_N(x,t)$ can be written in terms of the cubic B- splines as trial functions:

$$U_N(x,t) = \sum_{i=-1}^{N+1} \phi_i(x)\delta_i(t) \tag{4}$$

where δ_i's are unknown, time dependent quantities to be determined from the boundary and cubic B-spline collocation conditions. Each cubic B-spline covers four elements so that each element $[x_i, x_{i+1}]$ is covered by four cubic B-splines. For this problem, the finite elements are identified with the interval $[x_i, x_{i+1}]$

and the elements knots x_i, x_{i+1}. Using the nodal values $U_i, U_i^{'}$ and $U_i^{''}$ are given in terms of the parameter δ_i by:

$$
\begin{aligned}
U_i &= U(x_i) = \delta_{i-1} + 4\delta_i + \delta_{i+1}, \\
U_i' &= U'(x_i) = \tfrac{3}{h}(-\delta_{i-1} + \delta_{i+1}), \\
U_i'' &= U''(x_i) = \tfrac{6}{h^2}(\delta_{i-1} - 2\delta_i + \delta_{i+1})
\end{aligned} \tag{5}
$$

and the variation of $U_N(x,t)$ over the typical element $[x_i, x_{i+1}]$ is given by

$$U_N(x,t)= \sum_{j=i-1}^{i+2} \delta_j(t)\phi_j(x) \ . \tag{6}$$

If we substitute the global approximation (4) and its necessary derivatives (5) into Eq. (1), we obtain the following set of the first order ordinary differantial equations:

$$\dot{\delta}_{i-1} + 4\dot{\delta}_i + \dot{\delta}_{i+1} + \frac{9Z_i}{2h}(-\delta_{i-1} + \delta_{i+1}) - 6\frac{\mu}{h^2}(\dot{\delta}_{i-1} - 2\dot{\delta}_i + \dot{\delta}_{i+1}) = 0 \tag{7}$$

where

$$Z_i = (\delta_{i-1} + 4\delta_i + \delta_{i+1})^2$$

and $\cdot$ denotes derivative with respect to time. If time parameters δ_i's and its time derivatives $\dot{\delta}_i$'s in Eq. (7) are discretized by the Crank-Nicolson formula and usual finite difference aproximation, respectively:

$$\delta_i = \frac{1}{2}(\delta^n + \delta^{n+1}), \qquad \dot{\delta}_i = \frac{\delta^{n+1} - \delta^n}{\Delta t} \tag{8}$$

we obtain a recurrence relationship between two time levels n and $n+1$ relating two unknown parameters δ_i^{n+1}, δ_i^n for $i = m-1, m, m+1$,

$$\gamma_{m1}\delta_{m-1}^{n+1} + \gamma_{m2}\delta_m^{n+1} + \gamma_{m3}\delta_{m+1}^{n+1} = \gamma_{m3}\delta_{m-1}^n + \gamma_{m2}\delta_m^n + \gamma_{m1}\delta_{m+1}^n \tag{9}$$

where

$$
\begin{aligned}
&\gamma_{m1} = (1 - EZ_m - \mu), \qquad \gamma_{m2} = (4 + 2M), \qquad \gamma_{m3} = (1 + EZ_m - M) \\
&m = 0, 1, \ldots, N, \qquad E = \tfrac{9}{2h}\Delta t, \qquad M = \tfrac{6}{h^2}\mu.
\end{aligned} \tag{10}
$$

For the first linearization (First Lin.), we suppose that the quantity U in the non-linear term U^2U_x to be locally constant. This is equivalent to assuming that in Eq. (7) all U's are equal to a local constant Z_i.

For the second linearization (Second Lin.), using first order difference formula for the time derivative of the U and Crank-Nicolson approximation for the space derivatives U_x and U_{xx} in Eq. (1) lead to

$$\frac{U^{n+1} - U^n}{\Delta t} + 3\frac{(U^2U_x)^{n+1} + (U^2U_x)^n}{2} - \mu\frac{U_{xx}^{n+1} - U_{xx}^n}{\Delta t} = 0. \tag{11}$$

Now, if we apply Rubin and Graves [17] linearization technique to Eq. (11)

$$(U^2U_x)^{n+1} = U^{n+1}U^nU_x^n + U^nU^{n+1}U_x^n + U^nU^nU_x^{n+1} - 2UU^nU_x^n$$

we obtain

$$\begin{aligned} U^{n+1} + 3\tfrac{\Delta t}{2}(U^{n+1}U^nU_x^n + U^nU^{n+1}U_x^n + U^nU^nU_x^{n+1}) - \mu U_{xx}^{n+1} \\ = U^n - 3\tfrac{\Delta t}{2}(U^2U_x)^n - \mu U_{xx}^n + 6\tfrac{\Delta t}{2}(U^nU^nU_x^n). \end{aligned} \tag{12}$$

The system (9) consists of $N+1$ linear equations including $N+3$ unknown parameters $(\delta_{-1}, \ldots, \delta_{N+1})^T$. To obtain a unique solution to this system, we need two additional constraints. These are obtained from the boundary conditions and can be used to eliminate δ_{-1} and δ_{N+1} from the system (9) which then becomes a matrix equation for the $N+1$ unknowns $d = (\delta_0, \delta_1, \ldots, \delta_N)^T$ of the form

$$A\mathbf{d^{n+1}} = B\mathbf{d^n}. \tag{13}$$

The matrices A and B are tridiagonal $(N+1)\times(N+1)$ matrices and so are easily solved . However, two or three inner iterations are applied to the term $\delta^{n*} = \delta^n + \frac{1}{2}(\delta^n - \delta^{n-1})$ at each time step to cope with the non-linearity caused by Z_i.

2.1. Initial state

The initial vector d^0 is determined from the initial and boundary conditions. So the approximation (4) must be rewritten for the initial condition

$$U_N(x,0) = \sum_{i=-1}^{N+1} \delta_i^0(t)\phi_i(x) \tag{14}$$

where the δ_i^0's are unknown parameters. We require the initial numerical approximation $U_N(x,0)$ satisfy the following conditions:

$$\begin{aligned} U_N(x,0) &= U(x_i,0), \qquad && i = 0,1,...,N \\ (U_N)_x(a,0) &= 0, && (U_N)_x(b,0) = 0. \end{aligned} \tag{15}$$

Thus, these conditions lead to matrix equation

$$Wd^0 = b \tag{16}$$

where

$$W = \begin{bmatrix} 4 & 2 & & & & & \\ 1 & 4 & 1 & & & & \\ & 1 & 4 & 1 & & & \\ & & & & \ddots & & \\ & & & & 1 & 4 & 1 \\ & & & & & 2 & 4 \end{bmatrix}$$

$$d^0 = (\delta_0, \delta_1, \delta_2, \ldots, \delta_{N-2}, \delta_{N-1}, \delta_N)^T$$

and

$$b = (U(x_0,0), U(x_1,0), U(x_2,0), \ldots, U(x_{N-2},0), U(x_{N-1},0), U(x_N,0))^T.$$

2.2. Stability analysis

The investigation of the stability of the approximation obtained by the algorithm will be based on the von Neumann theory in which the growth factor of a typical Fourier mode is defined as:

$$\delta_j^n = \hat{\delta}^n e^{ijkh}, \tag{17}$$

where k is the mode number and h is the element size. Thus the stability analysis is determined for the linearisation of the approximation obtained by the numerical scheme. Substituting the Fourier mode (17) into the linearised recurrence relationship (9) shows that the growth factor for mod k is

$$g = \frac{a - ib}{a + ib} \tag{18}$$

where

$$\begin{aligned} a &= 2 + M + (1 - M)\cos[hk], \\ b &= EZ_i \sin[hk]. \end{aligned} \tag{19}$$

The modulus of $|g|$ is 1, therefore the linearised scheme is unconditionally stable.

3. Numerical Examples and Results

Numerical results of the equation for the three test problems were obtained and all computations were executed on a pentium PC4 in the Fortran code using double precision arithmetic. The MEW Eq. (1) possesses only three following conservation laws:

$$\begin{aligned} I_1 &= \textstyle\int_a^b U dx \simeq h \sum_{J=1}^N U_j^n, \\ I_2 &= \textstyle\int_a^b U^2 + \mu (U_x)^2 dx \simeq h \sum_{J=1}^N (U_j^n)^2 + \mu \, (U_x)_{j,}^n \\ I_3 &= \textstyle\int_a^b U^4 dx \simeq h \sum_{J=1}^N (U_j^n)^4. \end{aligned} \tag{20}$$

which correspond to mass, momentum and energy respectively [10]. The accuracy of the method is measured by both the error norm L_2

$$L_2 = \left\| U^{exact} - U_N \right\|_2 \simeq \sqrt{h \sum_{J=0}^{N} \left| U_j^{exact} - (U_N)_j \right|^2}, \tag{21}$$

and the error norm L_∞

$$L_\infty = \left\| U^{exact} - U_N \right\|_\infty \simeq \max_j \left| U_j^{exact} - (U_N)_j \right|. \tag{22}$$

To implement the method, three test problems: motion of a single solitary wave, interaction of two solitary waves and the maxwellian initial condition will be considered.

4. Motion of a Single Solitary Wave

The solitary wave solution of the MEW Eq.(1) is given by

$$U(x,t) = A \sec h(k[x - x_0 - vt])$$

where $k = \sqrt{1/\mu}$, $v = A^2/2$. This solution corresponds to motion of a single solitary wave of magnitude A, initially centered at the position x_0 and propagating to the right side with a constant velocity v. The initial condition is

$$U(x,0) = A \sec h(k[x - x_0]).$$

For this problem the analytical values of the invariants are [14]

$$I_1 = \frac{A\pi}{k}, \quad I_2 = \frac{2A^2}{k} + \frac{2\mu k A^2}{3}, \quad I_3 = \frac{4A^4}{3k}. \tag{23}$$

The analytical values of the invariants are obtained from Eq. (1) as $I_1 = 0.7853982$, $I_2 = 0.1666667$, $I_3 = 0.0052083$. To compare our results with the earlier papers, parameters are taken as $\Delta t = 0.05$, $\mu = 1$, $x_0 = 30$, $A = 0.25$ and the interval $0 \leq x \leq 80$ is divided into elements of equal lenght $h = 0.1$. The simulation is run up to time $t = 20$, and the three invariants I_1, I_2 and I_3 and error norms L_2, L_∞ are listed for the duration of the simulation. In Table 1, we compare the values of the invariants and error norms obtained using the present method with different approximations and those of [2, 5, 7] at different times. As seen from the table, the error norms L_2 and L_∞ are found to be small enough and the quantities in the variants remain almost constant during the computer run. While for the first linearization, invariants I_1, I_2 and I_3 change by less than $0.03 \times 10^{-5}\%$, $5.48 \times 10^{-5}\%$, $0.33 \times 10^{-5}\%$ for the second linearization they change less than $0.02 \times 10^{-5}\%$, $5.50 \times 10^{-5}\%$, $0.30 \times 10^{-5}\%$ throught the run, respectively. Thus it is seen that the invariants remain satisfactorily constant. Figure 1 shows that the proposed method performs the motion of propagation

of a solitary wave satisfactorily, which moves to the right at a constant speed and preserves its amplitude and shape with increasing time as expected. The amplitude is 0.25 at $t = 0$ and located at $x = 30.6$, while it is 0.249880 at $t = 20$ and located at $x = 30.6$. The absolute difference in amplitudes at times $t = 0$ and $t = 20$ is 12×10^{-5} so that there is a little change between amplitudes. The error graph at $t = 20$ is given in Figure 2. As it is seen, the maximum errors occur around the central position of the solitary wave.
This problem is also considered for different values of the amplitude at $h = 0.1$ and $t = 0.01$. In Table 2, the error norms and the invariants are listed for $A = 0.25, 0.5, 0.75, 1$. A comparison with Ref. [2] shows that the present method provides better results in terms of the error norms L_2 and L_∞. Figure 3 shows the solutions of the single solitary wave with $h = 0.1, \Delta t = 0.01$ for different values of amplitude A at time $t = 20$. It is clear that the soliton moves to the right at a constant speed and almost preserves its amplitude and shape with increasing of time, as expected.

5. Interaction of Two Solitary Waves

Now we consider Eq. (1) together with boundary conditions $U \to 0$ as $x \to \pm\infty$ and the initial condition for all linearization techniques as

$$U(x,0) = \sum_{j=1}^{2} A_j \sec h(k[x - x_j])$$

where $k = \sqrt{1/\mu}$.
Firstly, we have studied the interaction of two positive solitary waves with the parameters $h = 0.1,\ \Delta t = 0.025,\ \mu = 1,\ A_1 = 1,\ A_2 = 0.5$, $x_1 = 15,\ x_2 = 30$ through the interval $0 \le x \le 80$. The analytical values can be found as follows [5]:

$$\begin{aligned} I_1 &= \pi(A_1 + A_2) = 4.7123889, \\ I_2 &= \tfrac{8}{3}(A_1^2 + A_2^2) = 3.3333333, \\ I_3 &= \tfrac{4}{3}(A_1^4 + A_2^4) = 1.4166667. \end{aligned} \tag{24}$$

The experiment was run from time $t = 0$ to time $t = 80$ to allow the interaction take place. In Figure 4, we show the interaction of two positive solitary waves at different times. It can be seen that at time $t = 5$ the wave with larger amplitude is to the left of the second wave with smaller amplitude. The larger wave catches up the smaller one as time increases. Interaction starts at about time $t = 25$, overlapping processes occurres between times $t = 25$ and $t = 40$ and the waves start to resume their original shapes after time $t = 40$. An oscillation of small amplitude trailing behind the solitary waves in Fig. 4(f) was observed. In order to see this oscillation the scale of Fig. 4(f) was magnified as shown in Fig 5. At time $t = 80$, for the first linearization the amplitude of the larger wave is 0.999694 at the point $x = 44.4$ whereas the amplitude of the smaller one is 0.510405 at the point $x = 34.7$. For the second linearization, the amplitude of the larger wave is 0.999716 at the point $x = 56.9$ whereas the amplitude of the smaller one is 0.498438 at the point $x = 37.7$. Table 3 compares the values of the invariants of the two solitary waves with the obtained results from the first and the second linearization. The absolute difference between the values of the invariants obtained by the first linearization at times $t = 0$ and $t = 80$ are $\Delta I_1 = 1.2 \times 10^{-6}$, $\Delta I_2 = 4 \times 10^{-7}, \Delta I_3 = 0$ whereas they are $\Delta I_1 = 1.1 \times 10^{-6}, \Delta I_2 = 7.8 \times 10^{-6}, \Delta I_3 = 8 \times 10^{-6}6$ for the second linearization. Secondly, for the solitary of amplitudes -2 and 1 to interact, we have chosen the region as $0 \leq x \leq 150$ while keeping all other parameters the same as given before. The experiment was run from time $t = 0$ to time $t = 55$ to allow the interaction take place. Figure 6 shows the development of the solitary wave interaction. As it is seen from the Figure 6, at $t = 0$ a wave with the negative amplitude is to the left of another wave with the positive amplitude. The larger wave with the negative amplitude catches up the smaller one with the positive amplitude as time increases. At $t = 55$, for the first linearization the amplitude of the smaller wave is 0.974353 at the point $x = 52.5$, whereas the amplitude of the larger one is -1.986150 at the point $x = 122.7$. It is found that the absolute difference in amplitudes is 0.256×10^{-1} for the smaller wave and 0.138×10^{-1} for the larger one. For the second linearization, the amplitude of the smaller wave is 0.973607 at the point $x = 52.5$, whereas the amplitude of the larger one is -1.988065 at the point $x = 123.6$. It is found that the absolute difference in amplitudes is 0.263×10^{-1} for the smaller wave and 0.119×10^{-1} for the larger one. The analytical invariants by using Eq.(1) can be found as $I_1 = -3.1415927$, $I_2 = 13.3333333$, $I_3 = 22.6666667$. Table 4 lists the values of the invariants of the two solitary waves with amplitude $A_1 = -2$ and $A_2 = 1$ in the region $0 \leq x \leq 150$. It can be seen that the values obtained for the invariants are satisfactorily constant during the computer run.

5.1. The Maxwellian initial condition

For this equation another initial value problem is the initial Maxwellian pulse that is used as the initial condition in solitary waves given by

$$U(x, 0) = e^{-x^2} \tag{25}$$

with the boundary condition

$$U(-20, t) = U_x(-20, t) = U(20, t) = U_x(20, t) = 0, t > 0.$$

As it is known Maxwellian initial condition (25) breaks up into a number of solitary waves depending on values of μ. So we have used various values for μ. During the run of algorithms, we have taken $h = 0.1$, $\Delta t = 0.01$. The computations are carried out for the cases of $\mu = 1, 0.5, 0.1, 0.05, 0.02$ and 0.005 . For $\mu = 1$, the Maxwellian initial condition develops into a pair of waves as indicated in Figure 7. One wave with the negative amplitude is to the left of the other wave with the positive amplitude. For $\mu = 0.5$, the Maxwellian initial condition does not cause development into a clean solitary wave. When $\mu = 0.1$, we observed one clean solitary wave. For $\mu = 0.05$, the state is two solitary waves. For $\mu = 0.02$ and 0.005 three and seven solitary waves are formed, respectively. The recorded values of the invariants I_1, I_2 and I_3 computed for both linerazation techniques are given in Table 5 and 6. It is observed that the obtained values of the invariants remain almost constant during the computer run.

6. Conclusions

In this paper, numerical solutions of the MEW equation based on the cubic B-spline finite element have been presented. Three test problems are worked out to examine the performance of the algorithms. The performance and accuracy of the method is shown by calculating the error norms L_2 and L_∞. For each linearization technique, the error norms are sufficiently small and the invariants are satisfactorily constant in all computer runs. The computed results show that the present method is a remarkably successful numerical technique for solving the MEW equation and advisable for getting numerical solutions of other types of non-linear equations.

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Figures and Tables

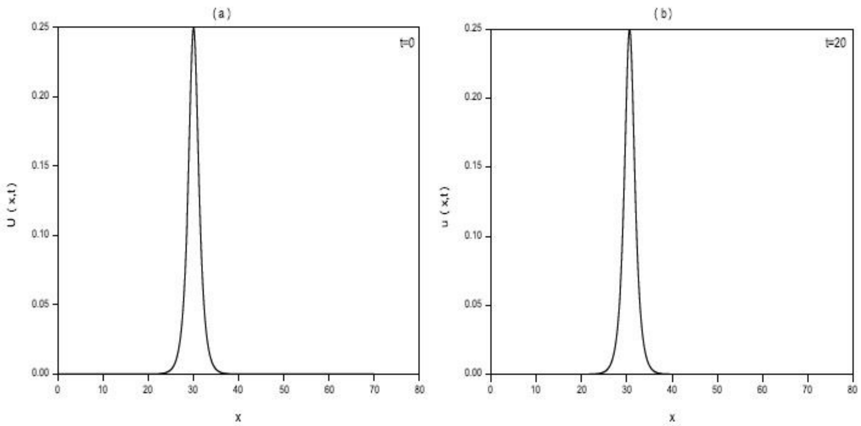

Figure 1. The motion of a single solitary wave with $h = 1,\ \Delta t = 0.05$ at (a) $t = 0$ and (b) $t = 20$.

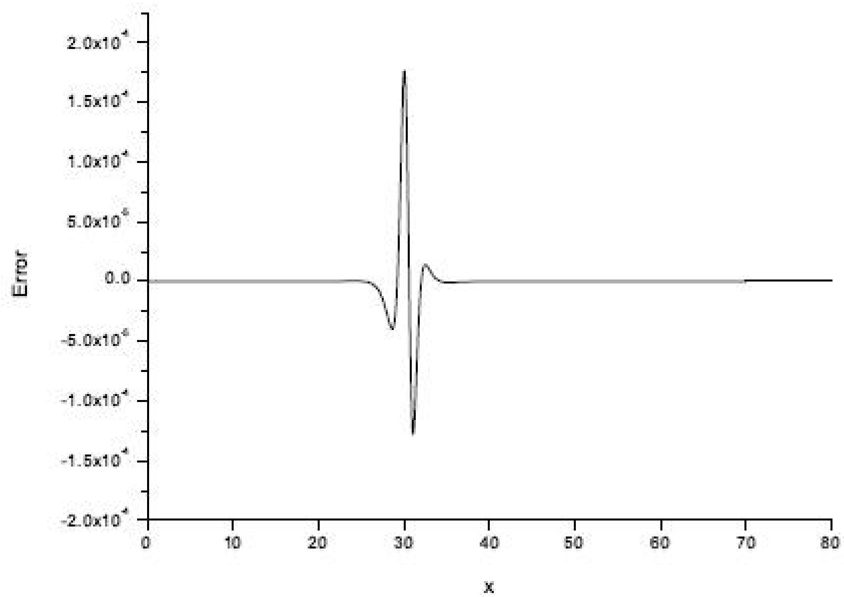

Figure 2. Error graph at $t = 20$.

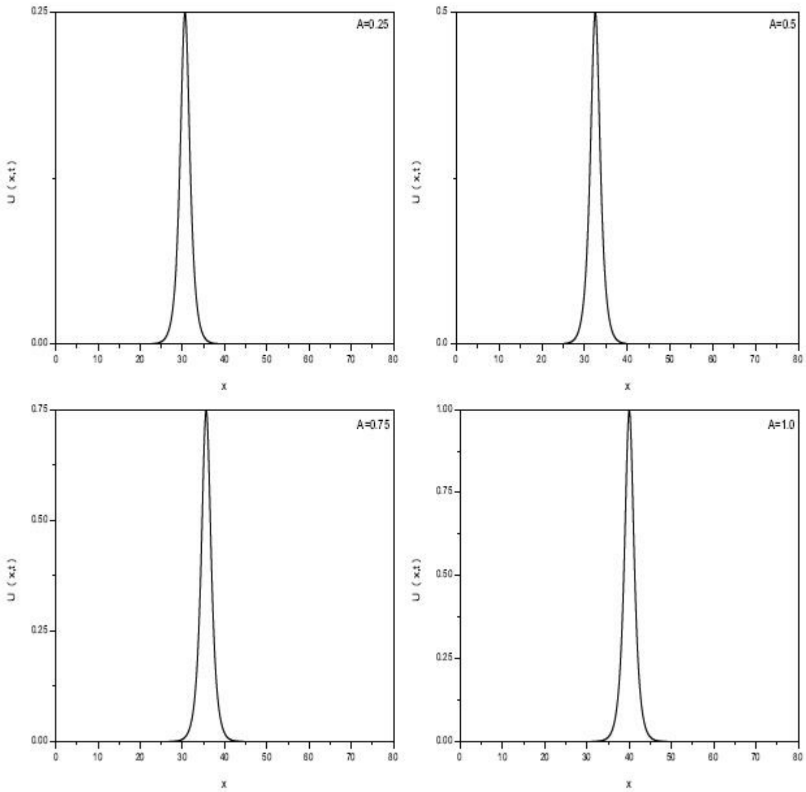

Figure 3. Single solitary wave solutions for various values of A at $t = 20$.

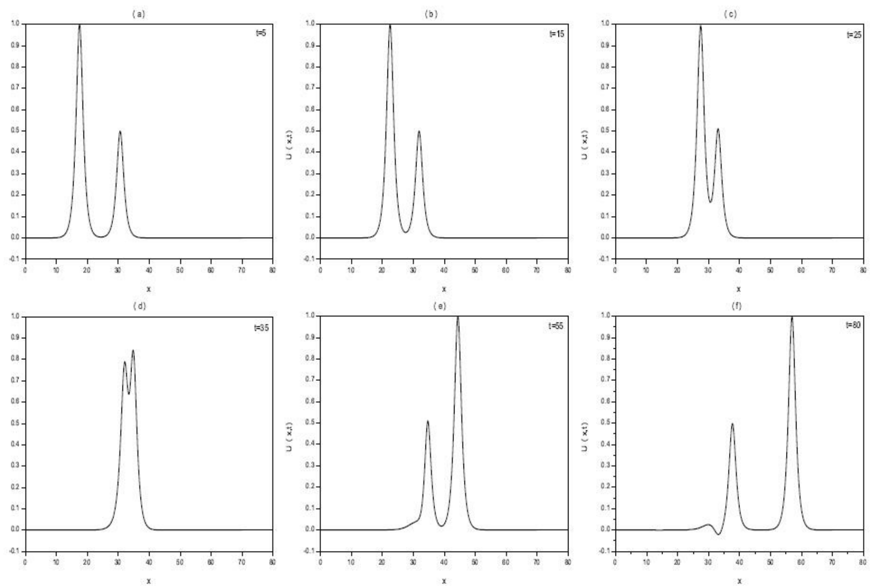

Figure 4. Interaction of two solitary waves at different times.

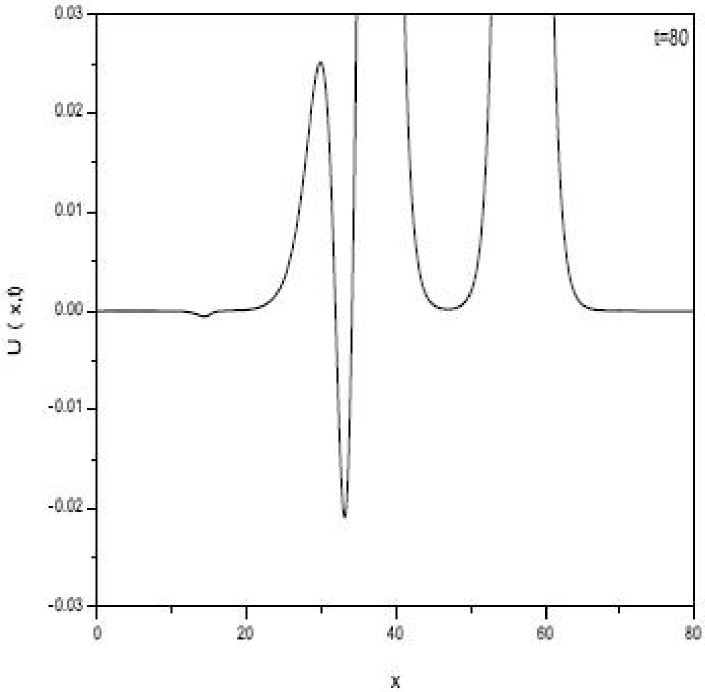

Figure 5. An expanded vertical scale of Fig.4(f) at t=80.

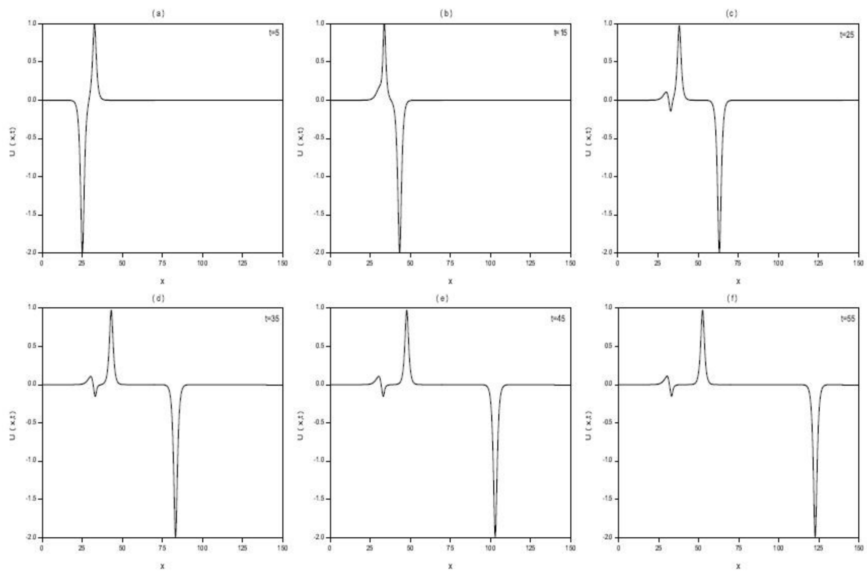

Figure 6. Interaction of two solitary waves at different times.

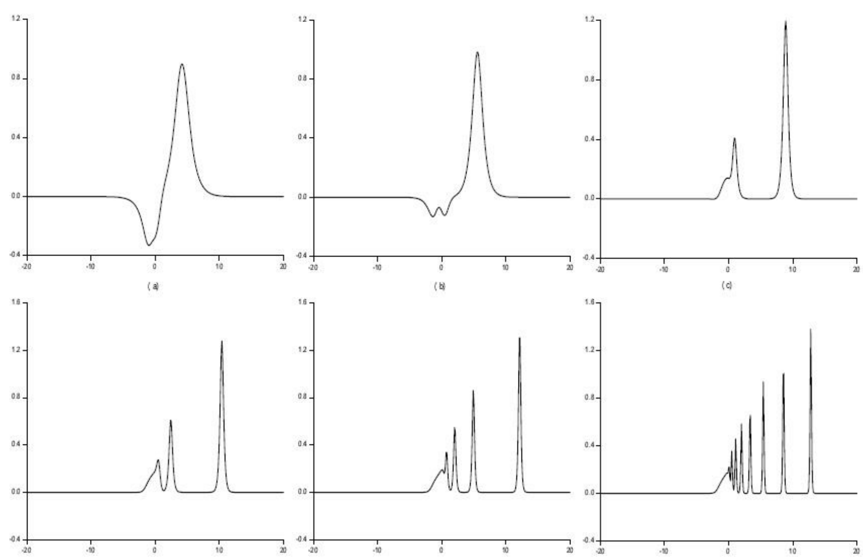

Figure 7. Maxwellianinitialcondition,stateattime $t = 12,\ a)\ \mu = 1,$ $b)\ \mu = 0.5,\ c)\ \mu = 0.1,\ d)\ \mu = 0.05,\ e)\ \mu = 0.02,\ f)\ \mu = 0.005$

Table 1. The invariants and the error norms for single solitary wave with $h = 0.1,\ \Delta t = 0.05,\ A = 0.25,\ 0 \leq x \leq 80$.

t	Linearization	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$
0		0.7853966	0.1666661	0.0052083	0.0000000	0.0000000
5		0.7853966	0.1666662	0.0052083	0.0447287	0.0423454
10	First	0.7853966	0.1666662	0.0052083	0.0890880	0.0867227
15		0.7853966	0.1666662	0.0052083	0.1327179	0.1316963
20		0.7853966	0.1666662	0.0052083	0.1752771	0.1764657
0		0.7853966	0.1666661	0.0052083	0.0000000	0.0000000
5		0.7853966	0.1666662	0.0052083	0.0447267	0.0423438
10	Second	0.7853966	0.1666662	0.0052083	0.0890842	0.0867198
15		0.7853966	0.1666662	0.0052083	0.1327126	0.1316924
20		0.7853966	0.1666662	0.0052083	0.1752706	0.1764596
20[2]		0.7853977	0.1664735	0.0052083	0.2692812	0.2569972
20[5]		0.7849545	0.1664765	0.0051995	0.2498925	0.2905166
20[7]		-	-	-	0.1958878	0.1744330

Table 2. The computed values I_1, I_2 and I_3 and the error norms L_2 and L_∞ for the single solitary wave with $x_0 = 30$, $h = 0.1$, $\Delta t = 0.01$ in the region $0 \leq x \leq 80$.

A	t	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$
0.25	0	0.7853966	0.1666661	0.0052083	0.00000000	0.0000000
	5	0.7853966	0.1666662	0.0052083	0.04471776	0.04233505
	10	0.7853966	0.1666662	0.0052083	0.08906601	0.08670116
	15	0.7853966	0.1666662	0.0052083	0.13268479	0.13166289
	20	0.7853967	0.1666662	0.0052083	0.17523269	0.17642205
	20[2]	-	-	-	0.2692249	0.2569562
0.5	0	1.5707932	0.6666646	0.0833330	0.0000000	0.0000000
	5	1.5707932	0.6666649	0.0833330	0.35052093	0.35289878
	10	1.5707932	0.6666656	0.0833330	0.65824902	0.65054805
	15	1.5707932	0.6666659	0.0833330	0.89807157	0.80335418
	20	1.5707931	0.6666660	0.0833330	1.06979673	0.86864227
	20[2]	-	-	-	1.82660590	1.4575680
0.75	0	2.3561897	1.4999953	0.4218734	0.0000000	0.0000000
	5	2.3561897	1.4999978	0.4218733	1.08833328	1.05417097
	10	2.3561897	1.4999985	0.4218733	1.70491172	1.33432195
	15	2.3561896	1.4999983	0.4218733	2.01264576	1.46558019
	20	2.3561896	1.4999982	0.4218733	2.24293300	1.62010840
	20[2]	-	-	-	4.3957110	3.0917930
1.0	0	3.1415863	2.6666583	1.3333283	0.00000000	0.0000000
	5	3.1415858	2.6666633	1.3333275	2.14753916	1.74396281
	10	3.1415852	2.6666624	1.3333268	2.87024724	2.07526179
	15	3.1415847	2.6666616	1.3333261	3.41524802	2.45685025
	20	3.1415842	2.6666609	1.3333253	3.98833508	2.84859636
	20[2]	-	-	-	8.2853140	5.6821310

Table 3. Invariants and error norms for single solitary wave with $A_1 = 1,\ A_2 = 0.5,\ h = 0.1,\ \Delta t = 0.05$

t	Linearization	I_1	I_2	I_3
0		4.7123733	3.3333253	1.4166643
10		4.7123744	3.3333216	1.4166642
20		4.7123744	3.3333202	1.4166640
30		4.7123744	3.3334406	1.4166576
40		4.7123749	3.3332759	1.4166614
50	First	4.7123751	3.3332328	1.4166646
55		4.7123750	3.3332427	1.4166646
60		4.7123748	3.3332663	1.4166645
70		4.7123745	3.3333083	1.4166643
80		4.7123745	3.3333257	1.4166643
0		4.7123733	3.3333253	1.4166643
10		4.7123744	3.3333303	1.4166630
20		4.7123744	3.3333274	1.4166614
30		4.7123744	3.3334217	1.4166397
40		4.7123748	3.3332640	1.4166490
50	Second	4.7123751	3.3332280	1.4166599
55		4.7123750	3.3332374	1.4166594
60		4.7123748	3.3332604	1.4166588
70		4.7123745	3.3333013	1.4166575
80		4.7123744	3.3333175	1.4166563

Table 4. Invariants and error norms for single solitary wave with $A_1 = -2,\ A_2 = 1,\ h = 0.1,\ \Delta t = 0.05$

t	Linearization	I_1	I_2	I_3
0		-3.1415739	13.3332816	22.6665313
5		-3.1415915	13.3220192	22.6214073
15		-3.1416695	13.2800001	22.4507282
25	First	-3.1417066	13.2588994	22.3620141
35		-3.1417376	13.2368786	22.2744329
45		-3.1417686	13.2150822	22.1879289
55		-3.1417997	13.1935069	22.1024801
0		-3.1415739	13.3332816	22.6665313
5		-3.1391878	13.3197925	22.6125339
15		-3.1325941	13.2800027	22.4661827
25	Second	-3.1278712	13.2544025	22.3595729
35		-3.1231851	13.2280040	22.2545970
45		-3.1185508	13.2019283	22.1511668
55		-3.1139673	13.1761691	22.0492444

Table 5. The invariants $I_1,\ I_2$ and I_3 obtained during the first linerazation technique for Maxwellian initial condition and different values of μ.

t	μ	I_1	I_2	I_3	μ	I_1	I_2	I_3
0	1	1.7724537	2.5066073	0.8862269	0.05	1.7724537	1.3159788	0.8862269
3		1.7724575	2.5066329	0.8862412		1.7724772	1.3108104	0.8879092
6		1.7724455	2.5083459	0.8862054		1.7722951	1.3095477	0.8872762
9		1.7724423	2.5089578	0.8861997		1.7720383	1.3090815	0.8865516
12		1.7724416	2.5089588	0.8861999		1.7717951	1.3086207	0.8858297
0	0.5	1.7724537	1.8799607	0.8862269	0.02	1.7724537	1.2783800	0.8862269
3		1.7724591	1.8794240	0.8862548		1.7722682	1.2675724	0.8938665
6		1.7724493	1.8803887	0.8862197		1.7707571	1.2633830	0.8876738
9		1.7724486	1.8804044	0.8862190		1.7690538	1.2599281	0.8814034
12		1.7724477	1.8803945	0.8862180		1.7674587	1.2566543	0.8753965
0	0.1	1.7724537	1.3786434	0.8862269	0.005	1.7724537	1.2595806	0.8862269
3		1.7724794	1.3755873	0.8867179		1.7706812	1.2537689	0.9975863
6		1.7724372	1.3754596	0.8866053		1.7561181	1.2181356	0.9105701
9		1.7724019	1.3752156	0.8864986		1.7428768	1.1841292	0.8076883
12		1.7723629	1.3750049	0.8863910		1.7355300	1.1793688	0.8151322

Table 6. The invariants $I_1,\ I_2$ and I_3 obtained during the second linerazation technique for Maxwellian initial condition and different values of μ.

t	μ	I_1	I_2	I_3	μ	I_1	I_2	I_3
0	1	1.7724537	2.5066073	0.8862269	0.05	1.7724537	1.3159788	0.8862269
3		1.7724579	2.5066333	0.8862417		1.7726434	1.3112290	0.8885770
6		1.7724462	2.5083441	0.8862055		1.7727156	1.3103771	0.8886312
9		1.7724431	2.5089549	0.8861997		1.7727126	1.3103199	0.8885918
12		1.7724425	2.5089559	0.8861999		1.7722229	1.3102667	0.8855532
0	0.5	1.7724537	1.8799607	0.8862269	0.02	1.7724537	1.2783800	0.8862269
3		1.7724603	1.8794269	0.8862569		1.7735172	1.2701095	0.8988266
6		1.7724515	1.8803898	0.8862219		1.7737897	1.2689727	0.8987264
9		1.7724516	1.8804063	0.8862220		1.7738490	1.2685999	0.8987989
12		1.7724515	1.8803973	0.8862220		1.7739619	1.2681353	0.8981215
0	0.1	1.7724537	1.3786434	0.8862269	0.005	1.7724537	1.2595806	0.8862269
3		1.7725113	1.3756928	0.8868511		1.7810616	1.2617060	0.9784616
6		1.7725176	1.3756383	0.8868441		1.7814725	1.2565368	0.9844928
9		1.7725309	1.3754676	0.8868429		1.7827076	1.2588371	0.9925560
12		1.7725405	1.3753301	0.8868407		1.7812130	1.2467044	0.9383488