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The Cauchy Problem for a Class of Nonlocal Boussinesq Type Equations

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Abstract

In this study the following Cauchy problem is considered:

$$\begin{aligned} &u_{tt} - u_{xx} - Lu_{xx} = (g(u))_{xx}, \quad x \in \mathbb{R}, \quad t > 0, \\ &u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x), \end{aligned}$$

where g is a sufficiently smooth nonlinear function and L is the linear operator defined by

$$\mathcal{F}\left(Lv\right)(\xi) = l\left(\xi\right)\mathcal{F}v\left(\xi\right).$$

Here $\mathcal{F}$ denotes the Fourier transform with respect to variable x and $l(\xi)$ is the Fourier transform of the kernel function associated with L. If $l\left(\xi\right)$ is a polynomial, then L is a differential operator. As a special case, if $l\left(\xi\right) = \xi^2$, then the equation under investigation turns out to be the Boussinesq equation. For non-polynomial functions $l\left(\xi\right)$, the equation under investigation is of nonlocal type. In our research, for a general class of kernel functions, we prove local existence of solutions of the Cauchy problem with initial data in suitable Sobolev spaces and establish the conditions for global existence and finite-time blow-up of solutions.

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Epidemic Models and Their Applications

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Abstract

The Susceptible-Infected-Recovered (SIR) epidemic model is a nonlinear two dimensional dynamical system characterized by a single parameter called the "basic reproduction number". This simplest model is representative of the spread of epidemics in a society where the total population is constant, the characteristic of the disease is time independent and no vaccination policy is applied. In the typical case, the ratio of the of susceptible individuals fall from a value S_0 close to 1 to a final value S_f while the ratio of recovered individuals rise from zero to $1 - S_f$. The basic SIR model is applied to H1N1 epidemic data in 2009-2010 in Istanbul [1] and in various European countries [2] and the dependency of the basic reproduction number to demographic and geographic parameters is modeled. The sharp passage from the level zero to the level final level R_f suggest the modeling of phase transitions by the SIR model. The sol-gel transition for alginate solutions with different concentrations is modeled as a SIR dynamical systems and it is shown that the phase transitions of "classical" and "percolation" type fall into different regions in the parameter-initial condition space [3].

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Bi-Hamiltonian and Bi-presymplectic Representations of Integrable ODE's

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Abstract

I will present the notion of bi-Hamiltonian representation and bi-presymplectic representation of Liouville integrable nonlinear Hamiltonian ODE's. Then I will explain why this additional structure of considered systems is sufficient for it's integration by quadratures.

Variable Parametric Quantum Oscillators and Exactly Solvable Schrödinger-Burgers Equations

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Abstract

We obtain exact solutions of a Madelung fluid model with dissipation, which is linearazible in the form of Schrödinger equation with time variable parameters. The corresponding Schrödinger-Burgers equation for complex velocity is solved by a generalized Cole-Hopf transformation and the dynamics of the pole singularities is described explicitly. In particular, we discuss and give exact solutions for variable parametric Madelung models related with the Caldirola-Kanai oscillator, and with the classical Sturm-Liouville problems for the orthogonal polynomials.

Dirichlet Problem for Generalized Inhomogeneous Polyharmonic Equation in an Annular Domain

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Abstract

In this talk, we will investigate the solvability of the Dirichlet problem in a ring domain for elliptic linear complex partial differential equations with main parts are polyharmonic operators. First, using the iteration of harmonic Green functions, we give higher order Green functions as fundamental solutions of the homogeneous problems. Secondly, we introduce some classes of operators related with Dirichlet problems with basic properties. Next, we transform the original problems into equivalent singular integral equations. Finally, solvability of the problems are discussed by defining the adjoint problems and using Fredholm alternative.

Nonlinear Modulation of In-Plane Extensional Waves in a Hyperelastic Plate

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Abstract

By employing the asymptotic perturbation methods previously used in the fields of fluid mechanics, plasma physics, etc., to investigate the propagation of weakly nonlinear waves, several problems related with the propagation of nonlinear dispersive elastic waves are examined [1-4]. The propagation of small, but finite amplitude long extensional waves in a nonlinear elastic plate of uniform thickness is considered in [5]. Here the asymptotic solution of a boundary value problem that is modeling the propagation of long extensional waves is built by a perturbation method called the method of multiple scales. With this method, the balance between the nonlinearity and dispersion yields a Korteveg-de Vries (KdV) equation for the asymptotic wave field.

In the present work, asymptotic analysis is applied to the same boundary value problem in [5] for nonlinear wave modulation. In the analysis, the weak nonlinearity and dispersion is balanced and it is concluded that the nonlinear modulation of external waves is characterized by a nonlinear Schrödinger (NLS) equation asymptotically. By taking into account the known properties of the solution of NLS equation [7,8], the behaviour of obtained asymptotic solutions of our problem as the existence of soliton solution are investigated for various nonlinear material properties of the plate. For this purpose, change of multiplication of coefficient functions of NLS equation according to dimensionless wave number is obtained with fixed linear material properties of the plate.

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Positive Solutions of the Second Order Semipositone m-point Boundary Value Problem on Time Scales

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Abstract

Let $\mathbb{T}$ be any time-scale(a nonempty closed subset of $\mathbb{R}$) with $a, b \in \mathbb{T}$. In this work, we are concerned with the following second order semipositone m-point boundary value problem

$$
\begin{aligned}
&-[p(t)u^{\triangle}(t)]^{\nabla} + q(t)u(t) = \lambda f(t, u(t)), \qquad t \in \mathbb{T}_k^k,\\
&\alpha u(\rho(a)) - \beta u^{[\triangle]}(\rho(a)) = \sum_{i=1}^{m-2} \alpha_i u(\xi_i),\\
&\gamma u(b) + \delta u^{[\triangle]}(b) = \sum_{i=1}^{m-2} \beta_i u(\xi_i),
\end{aligned}
$$

By using Krasnoselskii fixed point theorem, we present the existence of at least one or two positive solutions for the second order semipositone m-point boundary value problem on time scales. We emphasize that the nonlinear term f may take a negative value. As an application, we also give some examples to illustrate our results.

Symbolic Computation of Perturbation-Iteration Solutions for Differential Equations

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Abstract

An algorithm for the symbolic computation of perturbation-iteration solutions of nonlinear differential equations will be presented. In the algorithm the number of correction terms in the perturbation expansion (n) and the number of Taylor expansion terms (m) with $n <= m$ can be arbitrary. The steps of the algorithm will be illustrated on a Bratu type initial value problem. The algorithm has been implemented in *Mathematica*, a leading computer algebra system. The package *PerturbationIteration.m* will be briefly discussed.

Complex Solutions of the Finite Toda Lattice

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Abstract

In this talk, we construct complex solutions of the finite Toda lattice by use of the inverse spectral method. The corresponding Lax operator is a finite complex Jacobi matrix. The concept of spectral data for finite complex Jacobi matrices is introduced and a solution of the inverse spectral problem is presented.

Iterative Operator Splitting Method to Solute Transport Model: Analysis and Application

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Abstract

In this paper, iterative operator splitting method is used to solve numerically the transient two dimensional (2D) solute transport problem in ground water. The method is based on first splitting the complex problem (full advection-diffusion equation) into simpler sub-problems (advection and diffusion). Then each sub-equation is combined with iterative schemes. The consistency of the proposed method is investigated. The stability analysis of the iterative splitting is discussed in the sense of von Neumann for two dimensional cases. It is seen that the von Neumann analysis achieves the linear stability criteria. The numerical results show that the iterative splitting method provides more accurate results than other classical splitting methods.

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Some Comments on Recursion Operators

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Abstract

Recursion operators are also a Lax operator of the evolutionary type of nonlinear pdes. We discuss that such Lax operators are not suitable for the application of the Gelfand-Dikii formalism with the standard R-matrix.

Higher Symmetries and Effective Integrability Conditions for Quad Graph Equations

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Abstract

Generalized symmetry integrability test for discrete equations on the square lattice is studied. Integrability conditions are discussed. A method for searching higher symmetries (including non-autonomous ones) for quad graph equations is suggested based on characteristic vector fields.

Intial Boundary Value Problems for Nonlinear Equations of Fluid Flows in Prorous Media

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Abstract

We study the initial boundary value problems for Forchheimer and convective Brinkman û Forchheimer Equations.The problems of global solvability, structura stability and long time behavior of solutions of these problems will be discussed.

A Numerical Solution of the MEW Equation

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Abstract

In this study, modified equal width wave (MEW) equation is solved numerically by the Subdomain finite element method using quartic B-spline functions. Test problems including the motion of a single solitary wave and interaction of two solitary waves are studied to validate the suggested method. Accuracy and efficiency of the proposed method are discussed by computing the numerical conserved laws and L_2 , L_∞ error norms. The obtained results show that this method is a successful numerical technique for solving the modified equal width wave(MEW) equation. A linear stability analysis of the scheme shows it to be unconditionally stable.

We will consider the following one dimensional modified equal width wave (MEW) equation

$$U_t + 3U^2U_x - \mu U_{xxt} = 0, \tag{1}$$

with the boundary conditions

$$\begin{array}{lll} U(a,t) = 0, & U(b,t) = 0, & \\ U_x(a,t) = 0, & U_x(b,t) = 0, & t > 0, \end{array} \tag{2}$$

and the initial condition

$$U(x,0) = f(x) \quad a \leq x \leq b$$

where t is time, x is the space coordinate, μ is a positive parameter, $U(x,t)$ is wave amplitude and $f(x)$ is a prescribed function. The equal width wave (EW) equation, which is an alternative description of the nonlinear dispersive waves to the more usual Korteweg- de Vries (KdV) equation is a model nonlinear partial differential equation used for the simulation of one-dimensional non-linear waves propagating in dispersive media. Benjamin, Bona and Mahoney [1] proposed the regularized long wave (RLW) equation to be a model for the same physical phenomena equally well as the KdV equation. MEW equation, which we discuss here, is related with the modified regularized long wave (MRLW)equation [2] and modified Korteweg-de Vries (MKdV) equation [3] and is based upon the equal width wave (EW) equation. This equation has solitary wave solutions with both positive and negative amplitudes, all of which have the same width. The MEW equation is a non-linear wave equation with cubic nonlinearity with a pulse-like solitary wave solution [4]. Analytical solutions of the MEW equation are known with only a restricted set of boundary and initial conditions. Therefore, many numerical methods have been used for solving the MEW equation with various boundary and initial conditions. Wazwaz [5] investigated the MEW equation and two of its variants by the tanh and the sine-cosine methods. Zaki[6, 7] considered the solitary wave interactions for the MEW equation by Petrov-Galerkin method using quintic B-spline finite elements and obtained the numerical solution of the EW equation by using the least-squares method. Variational iteration method is introduced to solve the MEW equation by Junfeng Lu [8]. Esen[9, 10]applied a lumped Galerkin method based on quadratic B-spline finite elements have been used for solving the EW and MEW equation. A. Esen and S. Kutluay[11] studied a linearized implicit finite difference method in solving the MEW equation. Saka[12] proposed algorithms for the numerical solution of the MEW equation using quintic B-spline collocation method. T. Geyikli and S. Battal Gazi Karakoç[13, 14] solved the MEW equation by a collocation method using septic B-spline finite elements and using a Petrov-Galerkin finite element method with weight functions quadratic and element shape functions are cubic B-splines. D. J. Evans and K. R. Raslan [15] studied the generalized EW equation by using collocation method based on quadratic B-splines to obtain the numerical solutions of a single solitary waves, and the birth of solitons. In this paper, we solve the MEW equation numerically by Subdomain finite element method using quartic B-splines. The performance and accuracy of the proposed method have been tested on two problems: the motion of a single solitary wave and the interaction of two solitary waves. A linear stability analysis based on a Fourier method shows that the numerical scheme is unconditionally stable.

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High-Frequency Scattering of Acoustic Waves: A Convergent Algorithm

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Abstract

The scattering of acoustic plane waves by a sound soft obstacle $D \subset \mathbb{R}^2$ leads to exterior problems for the Helmholtz equation, i.e.

$$\begin{cases} \Delta u^i(x) + k^2 u^i(x) = 0, \quad x \in \mathbb{R}^2, \\ \Delta u^s(x) + k^2 u^s(x) = 0, \quad x \in \mathbb{R}^2 \backslash \bar{D}, \\ u^s = -u^i, \quad \text{on } \partial D, \\ \frac{x}{|x|} \cdot \nabla u^s(x) - iku^s(x) = o\left(|x|^{-1/2}\right), \text{ as } |x| \to \infty. \end{cases}$$

where $u^i(x)$ is the incident wave impinging on ∂D and $u^s(x)$ is the scattered wave. The classical numerical methods developed for the solution of such problems give rise to number of degrees of freedom that (at best) increase linearly with increasing wave number k. Accordingly, they are not suitable for high-frequency ($k \gg 1$) simulations. In this talk, considering two dimensional convex obstacles, we will present a new algorithm for the solution of high-frequency scattering problems. Our approach is based upon utilization of well-posed integral equation formulation of the scattering problem, and a non-standard Galerkin approximation space adopted to the known asymptotic properties (boundary layers) of the solution. As our main convergence result will display, it requires only a minor increase (k^ϵ for any $\epsilon > 0$) in the number of degrees of freedom to maintain a fixed accuracy. Finally, we will demonstrate a comparison of our theoretical findings with actual numerical results.

Symmetric Iterative Splitting Method for Non-Autonomous Systems

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Abstract

The iterative splitting methods have been extensively applied to solve complicated systems of differential equations. In this process we split the complex problem into several sub-problems, each of which can be solved sequentially. In this paper, we develop a symmetric iterative splitting scheme based on the magnus expansion for solving non-autonomous problems. We also study its convergence properties by using the concepts of stability, consistency, and order. Several numerical examples are illustrated to confirm the theoretical results by comparing frequently used methods.

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Exact Solutions and Classification of Symmetry Algebras for Variable Coefficient Nonlinear Schrödinger Equations

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Abstract

Analytical solutions of variable coefficient nonlinear Schrödinger equations having four-dimensional symmetry groups which are in fact the next closest to the integrable ones occurring only when the Lie symmetry group is five-dimensional are obtained using two different tools. The first tool is to use one dimensional subgroups of the full symmetry group to generate solutions from those of the reduced ODEs (Ordinary Differential Equations), namely group invariant solutions. The other is by truncation in their Painleve expansions. We also intend to present partial results on classification of symmetry algebras of a Schrdinger type equation.

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Maximal Regularity Properties of Parameter General Elliptic Equation in Hilbert Valued Sobolev Spaces

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Abstract

In this study we shall consider in the whole spaces $\mathbb{R}^n$ the general elliptic equation $A(D,q)u(D,q) = f(D,q)$. i.e. we investigate that a unique solution $u \in W^{l,2}(\mathbb{R}^n; H)$ is in existence for each $f \in W^{l-s,2}(\mathbb{R}^n; H)$.

$$A(D,q) = \sum_{|\alpha|+\beta \leq s} a_{\alpha\beta} q^{\beta} D^{\alpha}$$

is a differential operator with constant complex coefficients depending on a complex parameter q which varies in the limits of a closed sector of complex plane, with vertex at the origin of coordinates: [1]

$$Q = \{q : \alpha \leq argq \leq \beta\}$$

Existence and uniqueness (i.e. maximal regularity) of the solution of general boundary value problem with a parameter whose components are constant complex coefficients defined in n dimensional Euclidean space for $q = 2$ had been investigated by M.S. Agranovic ve M.I Vishik for space $W^{(l,2)}(\mathbb{R}^n)$ which is complex valued Sobolev space [2]. We have also investigated maximal regularity of the problem $W^{(l,2)}(\mathbb{R}^n; H)$, in which H is any Hilbert speace. In this study, fist of all boundedness of the elliptic operator A(D,q) was proved by using generalized Fourier transformation and extension theorem [3] [4] , then we can estimate

$$|||u|||_{W^{l,2}} \leq |||Au|||_{W^{l-s,2}}$$

where the constants C neither depend on u nor on q. In other words maximal regularity of the problem was proved by using Fourier transformation and extension theorem and Plancherel theorem. In conclusion, the maximal regularity

of the general elliptic problem depending on a parameter q, while its order is s, is shown under the certain conditions [5].

Keywords Elliptic operators, Sobolev space, Hilbert space, maximal regularity.

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2 + 1 KdV(N) Equations

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Abstract

We present some nonlinear partial differential equations in 2 + 1-dimensions derived from the KdV Equation and its symmetries. We show that all these equations have the same 3-soliton structures. The only difference in these solutions are the dispersion relations. We also show that they posses the Painleve property.

Toda Equation(its discretization on $\mathbb{R}$)

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Abstract

A differential-q-difference and differential-difference analogue of Toda equation is constructed by the concept of regular grain structures on $\mathbb{R}$. Using Hirota direct method via dependent variable transformation the one and two soliton solutions for these equations are derived and plotted to discuss the properties of solitons.

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On Trivial (de)compositions of Compatible Pairs of Hamiltonian Operators

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Abstract

It is proven that the so-called trivial compositions of a compatible pair of Hamiltonian operators form a set of mutually compatible Hamiltonian operators. Using this information some recently obtained higher order scalar Hamiltonian operators are shown to be related to known Hamiltonian operator pairs.

Localization, Smoothness, and Convergence to Equilibrium for a Thin Film Equation

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Abstract

We investigate the long-time behavior of weak solutions to the thin-film type equation

$$v_t = (xv - vv_{xxx})_x,$$

which arises in the Hele-Shaw problem. We estimate the rate of convergence of solutions to the Smyth-Hill equilibrium solution, which has the form $\frac{1}{24}(C^2 - x^2)_+^2$, in the norm

$$|||f|||_{m,1}^2 = \int_{\mathbf{R}} (1 + |x|^{2m})|f(x)|^2 dx + \int_{\mathbf{R}} |f_x(x)|^2 dx.$$

We obtain exponential convergence in the $|||\cdot|||_{m,1}$ norm for all m with $1 \leq m < 2$, thus obtaining rates of convergence in norms measuring both smoothness and localization. The localization is the main novelty, and in fact, we show that there is a close connection between the localization bounds and the smoothness bounds: Convergence of second moments implies convergence in the H^1 Sobolev norm. We then use methods of optimal mass transportation to obtain the convergence of the required moments. We also use such methods to construct an appropriate class of weak solutions for which all of the estimates on which our convergence analysis depends may be rigorously derived. Though our main results on convergence can be stated without reference to optimal mass transportation, essential use of this theory is made throughout our analysis. This is a joint work with Eric A. Carlen.

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The Dynamic Cauchy Problem in Banach Spaces

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Abstract

In this paper we focus on the existence of solution and Carathéodory solution of the dynamic Cauchy problem

$$\begin{array}{l} x^{\Delta}(t) = f(t, x(t)) \\ x(0) = x_0 \end{array} \quad , \quad t \in I_a \subset \mathbb{T},$$

in Banach spaces.
We were, particular, motivated by interesting papers of [1, 2, 3]. Authors present results which guarantee the existence of one or more solutions to particular cases of the dynamic Cauchy problem (DCP). The result of this paper extends those results. A Mönch fixed point theorem [4] and the techniques of the theory of the measure of noncompactness which is initiated by Banaś [5] are used to prove the existence of solution of DCP. By imposing some conditions expressed in terms of the measure of noncompactness on f, we define an operator over the Banach space (the space of rd-continuous functions from a time scale interval to a Banach space), whose fixed points are solutions of DCP.

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Cartan Matrices and Integrable Lattice Toda Field Equations

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Abstract

We study differential-difference integrable exponential type systems, corresponding to the Cartan matrices of semi-simple or affine Lie algebras. The integrability of the systems in terms of x- and n-integrals is discussed. In particular for systems corresponding to the algebras A_2, B_2, C_2, G_2 the complete sets of integrals in both directions are found.

Existence and Stability of Solitary Wave Solutions for a Class of Nonlocal Nonlinear Equations

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Abstract

The present study is concerned with the existence and stability of solitary wave solutions $u = \phi(x - ct)$ of a general class of nonlocal nonlinear equation of the form

$$u_{tt} = [\beta * (u + g(u))]_{xx}, \quad x \in \mathbb{R}, \quad t > 0, \tag{1}$$

where the subscripts denote partial differentiation, $u = u(x,t) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a real valued function, g is a nonlinear function of u and

$$(\beta * f)(x) = \int_{\mathbb{R}} \beta(x - y) f(y) dy$$

denotes the convolution of β and f. The kernel function $\beta(x)$ is assumed as an integrable function whose Fourier transform, $\hat{\beta}(\xi)$, satisfies

$$C_2^2 (1 + \xi^2)^{-r/2} \leq \hat{\beta}(\xi) \leq C_1^2 (1 + \xi^2)^{-r/2}, \quad r \geq 2.$$

The nonlocal nonlinear equation (1) is introduced in [1] as a model for the description of one dimensional bi-directional motion in nonlocally and nonlinearly elastic infinite medium. Using the Concentration- Compactness Principle [2], we establish the existence and stability of solitary wave solutions for the nonlocal equation.

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