

## Safety Analysis of Constructions

Jamalov Jasarat Aladdin\*

*Candidate of technical sciences, Scientific Research Institute of Construction and Architecture of Azerbaijan*

### Abstract

Safety analysis method using the theory of tolerance in designing engineering constructions is suggested. For the distribution curves  $x, y, \dots$ , the normal distribution law is accepted. The method allows to reveal some strength reserves and provides equal safety characterized by the factor  $\gamma$ .

Different factors that allow to consider the random character of loads operating on the construction should be taken into account for analysis of constructions. Accuracy of strength analysis and deformability of constructions is resolving for revision of safety factor quantities.

**Keywords:** Random, coefficient, stress, probability, construction, safety

## Yapıların Güven Analizi

### Özet

Yapı tasarımı mühendisliğinde kullanılan tolerans teorisinde güven analizi yapılması önerilmektedir. Metot, bazı güç kaynaklarını ortaya çıkarır ve  $\gamma$  faktörü ile karakterize edilen eşdeğer güvenliği sağlar. X, Y gibi dağılım eğrileri için normal dağılım kuralını kabul eder.

Yapı üzerinde yükleme işlemini rastgele karakterize etmeye müsaade eden farklı faktörler yapı analizinde dikkate alınmalıdır. Güç analizinin doğruluğu ve yapıların şekil değiştirilebilirliği güvenlik faktörü niceliğinin gözden geçirilebilmesini gerektirir.

**Anahtar Kelimeler:** Rasgele, katsayısı, stres, olasılık, inşaat, güvenlik

---

\*e-mail: jasarat.jamalov@fhn.gov.az

## 1. Introduction

Different factors by means of which the random character of loads acting on the construction and properties of the construction's material is grounded, should be taken into account while designing constructions. The accuracy matters of strength and deformability analysis of constructions are decisive for revising safety factor quantities (load and homogeneity factors).

The factors that influence of the accuracy of analysis are related with imperfections of technological processes of making constructions and materials for them that reduces to variability of their properties. This reduces to load increase compared with their design value. Improvement of technological processes of making and erection of constructions, standardization, right operation may essentially diminish inclination of design quantities and enable to lower the safety factors and reduce to saving in material.

Design methods proceeding from the way on admissible stresses don't enable to take into account operational and technological factors. Therewith, normalization of admissible stresses and design loads reduces to discrepancy between the accuracy of accomplished calculations and approximate assignment of safety factors without sufficient reasons [1-5].

The more perfect method using the limit state analysis also doesn't solve this problem completely since the significant load-carrying reserves are not used as a result of superfluous careful introduction of homogeneity and overload factors in their very unfavorable combination that in reality never may arise [6-9].

Probability of passing the adopted corrections the bounds determined on the base of design experience and operation practice to the factors found in the statistical way show that in the presence of only one design quantity deviating from the mean value is very small (~1/700, in the presence of three standard admissible deviations and normal distribution law).

## 2. Basic material

### 2.1. Basic dependence

Safety analysis method based on the use of variation statistics [10-14], actually proceeds from the theory of tolerance in designing of engineering constructions taking into account tolerance both geometric sizes and in strength properties of materials and in quantities of loads. As a result, it is defined a safety factor on which the loads reducing to limit state should be decreased in order to provide safety of the latter's operation.

The construction's undestruction condition may be written as follows:

$$R(x, y, \dots) = r - q > 0 \quad (1)$$

where  $x, y, \dots$  are design quantities that have some distribution curves;  $r$  is construction's strength measured in some units (for instance, by the material's strength limit in MPa).  $q$  is load on the construction measured in the units of the same measurement (MPa) in the dangerous section caused by the external forces. The distribution curve  $R$  is determined by the distribution curves  $x, y, \dots$ , therewith the area of the distribution curve  $R$  in the range  $R < 0$  should be equal to the preassigned very small quantity.

For simplifying the calculation process, the distribution curves  $x, y, \dots$  are approximately accepted as normal distribution curves, the function  $R(x, y, \dots)$  expands in series of powers  $x, y, \dots$  in the domain of their mean values and only linear terms remain in this series. In this case, the distribution curve  $R$  is obtained also in a normal form and its center  $m_R$  is determined by the formula:

$$m_R = R(m_x, m_y, \dots), \quad (2)$$

where  $m_x, m_y, \dots$  is a centre of the distribution  $x, y, \dots$  and the standard  $S_R$  is determined by the formula:

$$S_R^2 = S_x^2 \frac{\partial R}{\partial x}(m_x, m_y, \dots) + S_y^2 \frac{\partial R}{\partial y}(m_x, m_y, \dots) + \dots \quad (3)$$

moreover,  $X = m_x, Y = m_y, \dots$

The more accuracy, the less variability  $\frac{S_x}{m_x}, \frac{S_y}{m_y}, \dots$  of initial design quantities.

For applying this method to establish safe sizes of the construction, a formula that at some extent exactly reflects the structure's real work should be given. By this formula, the value of the breaking load or the desired size is calculated without any safety, i.e. ignoring the overload factors and homogeneity of the material, having taken mean design quantities. Then, using formula (3), by the standards of initial quantities, we find the standard of the desired quantity that is multiplied by the safety characteristics  $\gamma$  (the number of standards defining probability of passage through the limit state of the construction) and is added to the quantity found previously. Degree of approximation of such an approach is that the distribution curve of the calculated size is assumed to be a normal curve. Consider a numerical example in order to understand the gist of the suggested method.

## 2.2. Example 1

The section of the symmetric reinforcement of ferroconcrete column under eccentric compression should be taken for the following data:

Concrete B 15 (15 MPa);

Homogeneity factor  $K_0(R_b) = 0,85$  standard  $S(R_b) = 150 \frac{1 - 0,85}{3} = 7,5 \text{ kg} / \text{cm}^2$ , here

$\gamma = 3$  is the number of standards. The yield point of the reinforcement  $R_a^n = 210 \text{ MPa}$

Modulus of elasticity  $E_a = 2,1 \cdot 10^5 \text{ MPa}$

Homogeneity factor  $K_0(R_{sn}^\alpha) = 0,9$

Standard  $S(R_{sn}^\alpha) = 2100 \frac{1 - 0,9}{3} = 70 \text{ kg} / \text{cm}^2$

Acting loads:

Permanent load (density)

$$N_1 = 450 \text{ kN} ; \quad M_2 = 40 \text{ kNm}$$

Overload factor  $\gamma_f = 1,1$ . Standard  $S(N_1) = 450 \cdot \frac{1,1 - 1}{3} = 15 \text{ kN}$ . Temporary load (snow)

$$N_2 = 80 \text{ kN} ; \quad M_2 = 15 \text{ kNm} \quad \text{Overload factor } \gamma_f = 1,4$$

$$\text{Standard } S(N_2) = 80 \cdot \frac{1,4 - 1}{3} = 10,6 \text{ kN}$$

$$\text{Wind: } N_3 = 0 ; \quad M_3 = 60 \text{ kNm} \quad \text{Overload factor } \gamma_f = 1,4$$

$$\text{Standard } S(M_3) = 60 \cdot \frac{1,4 - 1}{3} = 8 \text{ kN}$$

The factor of bending moment increase taking into account increased flexibility of the column is accepted equal to 1,24 [15] i.e. exaggerated. Geometric parameters:  $h = 45 \text{ cm}$ ;  $b = 40 \text{ cm}$ ;  $h_o = 41,5 \text{ cm}$ ;

$$h_o - a' = 38 \text{ cm.}$$

Eccentricities with respect to the less compressed reinforcement equal:

$$e_1 = \frac{40 \cdot 100}{450} 1,24 + 22,5 - 3,5 = 30,02 \text{ cm}$$

$$e_2 = \frac{15 \cdot 100}{80} 1,24 + 22,5 - 3,5 = 37,75 \text{ cm}$$

Total bending moment:

$$\Sigma M = 450 \cdot 0,3002 + 80 \cdot 0,3775 + 60 \cdot 1,24 = 239,6 \text{ kNm}$$

The same normal force:

$$\Sigma N = 450 + 80 = 530 \text{ kN}$$

Determine limiting value of the compressed reinforcement area by the formula

$$A_a = \frac{1}{R_{sm} (h_o - a')} \left[ \Sigma N_i \ell_i + M - h_o \Sigma N_i \left( 1 - \frac{\Sigma N_i}{2 b h_o R_b} \right) \right] \quad (4)$$

Substituting the values of the parameters into this expression, we calculate:

$$A_a = \frac{239,63 \cdot 10^3}{210 \cdot 38} - \frac{41,5}{38} \cdot \frac{530 \cdot 10}{210} \left( 1 - \frac{530 \cdot 10}{2 \cdot 40 \cdot 41,5 \cdot 15} \right) = 5,38 \text{ cm}^2 \quad (5)$$

We calculated the area of the reinforcement  $A_a = 5,38 \text{ cm}^2$  ignoring the overload and homogeneity factors.

For determining the standard of the area  $A_a$  from formula (4) we have:

$$\frac{\partial A_a}{\partial N_i} = \frac{1}{R_{sn} (h_o - a^1)} \quad (6)$$

$$\frac{\partial A_a}{\partial R_b} = \frac{-(\sum N_i)^2}{2b(h_o - a^1)R_{sn}} \cdot \frac{1}{R_b^2} \quad (7)$$

$$\frac{\partial A_a}{\partial R_{sc}} = \frac{1}{(R_{sn})^2 (h_o - a^1)} \left[ \sum N_i \ell_i + M - h_o \sum N_i \left(1 - \frac{\sum N_i}{2bh_o \cdot R_b}\right) \right] = -\frac{A_a}{R_{sn}} \quad (8)$$

Substituting the numerical values into the expressions (5-8), we calculate:

$$\frac{\partial A_a}{\partial N_1} = \frac{1}{210 \cdot 38 \cdot 10} \left(30 - 41,5 + \frac{530 \cdot 10}{40 \cdot 15}\right) = -0,125 \cdot 10^{-4}$$

$$\frac{\partial A_a}{\partial N_2} = \frac{1}{210 \cdot 38} \left(37,75 - 41,5 + \frac{530 \cdot 10}{40 \cdot 15}\right) = -0,845 \cdot 10^{-4}$$

$$\frac{\partial A_a}{\partial M} = \frac{1,24}{210 \cdot 38 \cdot 10} = -0,155 \cdot 10^{-4}$$

$$\frac{\partial A_a}{\partial R_{sn}} = -\frac{5,38}{210 \cdot 10} = -0,256 \cdot 10^{-2}$$

We also define:

$$\frac{\partial A_a}{\partial R_b} = -\frac{(530 \cdot 10^2)^2}{2 \cdot 40 \cdot 38 \cdot 210 \cdot 10} \cdot \frac{1}{150^2} = -0,0195$$

Using formula (3), calculate:

$$S^2(A_a) = (15 \cdot 100)^2 \cdot (-0,125 \cdot 10^{-4})^2 + 1067^2 (0,845 \cdot 10^{-4})^2 + (8 \cdot 10^{-4})^2 \cdot (0,155 \cdot 10^{-4})^2 + (70)^2 \cdot (-0,256 \cdot 10^{-2})^2 + 7,5^2 \cdot 0,0195^2 = 1,596$$

$$S(A_a) = \sqrt{1,597} = 1,26 \text{ cm}^2$$

Necessary amount of the reinforcement for  $\gamma = 3$  :

$$A_a = 5,28 + 3 \cdot 1,28 = 9,16 \text{ cm}^2$$

Analysis by building code and rules (allowing for overload and homogeneity factor):

$$A_a = \frac{239,63 \cdot 10^3 \cdot 1,4}{210 \cdot 38 \cdot 0,9} - \frac{41,5}{38} \cdot \frac{(450 \cdot 1,1 + 80 \cdot 1,4)}{210 \cdot 0,9} \left(1 - \frac{(450 \cdot 1,1 + 80 \cdot 1,4) \cdot 10}{2 \cdot 40 \cdot 15 \cdot 0,85}\right) = 12,04 \text{ cm}^2$$

$$\text{The saving is } \frac{12,04 - 9,16}{12,04} \cdot 100 \% = 23,92 \% .$$

### 2.3. Example 2

It is required to choose the section of the reinforcement for T-shaped plain reinforced-concrete beam.

Given: height of the plain beam  $h = 70$  cm.

$$M_1 = 30 \text{ kNm} ; \gamma_{f_1} = 1,1 ; M_2 = 240 \text{ kNm} ; \gamma_{f_2} = 1,3$$

$$h_o - t_f / 2 = 69 \text{ cm}; \quad t_f \text{ is thickness of the flange of the beam.}$$

Characteristics of the materials are taken as in example 1:

Standards:  $S(R_{sn}) = 70 \text{ kg/cm}^2$  (see example 1)

$$S(M_1) = 30 \frac{1,1 - 1}{3} = 1 \text{ kNm}$$

$$S(M_2) = 240 \frac{1,3 - 1}{3} = 24 \text{ kNm}$$

Design formula:

$$A_a = \frac{\Sigma M_i}{R_{sn} (h_o - t_f / 2)}$$

Limiting value of  $A_a$  equals:

$$A_a = \frac{(30 + 240) \cdot 10^3}{210 \cdot 69} = 18,6 \text{ cm}^2$$

$$\frac{\partial A_a}{\partial M_1} = \frac{\partial A_a}{\partial M_2} = \frac{1}{R_{sn} (h_o - t_f / 2)} = \frac{1}{210 \cdot 10 \cdot 69} = 0,1449 \cdot 10^{-6}$$

$$\frac{\partial A_a}{\partial R_{sn}} = - \frac{1}{R_{sn}^2} \cdot \frac{\Sigma M}{(h_o - 0,5 t_f)} = - \frac{A_a}{R_{sn}} = \frac{18,6}{2100} = 0,885 \cdot 10^{-2}$$

$$S^2(A_a) = (1 \cdot 10^4)^2 \cdot (0,1449 \cdot 10^{-6})^2 + (24 \cdot 10^4)^2 \cdot (0,1449 \cdot 10^{-6})^2 + 70^2 (-0,885 \cdot 10^{-2})^2 = 0,3849$$

$$S(A_a) = 0,62 \text{ cm}^2$$

Necessary amount of the reinforcement for  $\gamma = 3$  :

$$A_a = 18,6 + 3 \cdot 0,62 = 20,46 \text{ cm}^2$$

The amount of the reinforcement calculated according to building codes:

$$A_a = \frac{M_1 \gamma_{f_1} + M_2 \gamma_{f_2}}{R_{sc} (h_o - t_f / 2)} = \frac{(30 \cdot 1,1 + 240 \cdot 1,3) 10^3}{210 \cdot 0,9 \cdot 69} = 26,45 \text{ cm}^2$$

The saving is:

$$\frac{26,45 - 20,46}{26,45} \cdot 100 \% \approx 22,62 \%$$

### 3. Conclusions

As is seen, the given calculation way allows to reveal some strength reserves and provides equal safety characterized by the definite number  $\gamma$ . The accuracy of the way under asymmetric distribution curves and great variability of design quantities are comparatively small but virtually, it may be considered acceptable. The accuracy increases while passing to more homogeneous materials and stable loads

### 4. References

- [1] Rzhantsin A.R., "Economical principle of safety analysis" Stroitel'naya mekhanika i raschet sooruzheniy. 1953, № 3.
- [2] Chiras A.A., "On rational design of elasticsplastic discrete systems under random loads". Reliability problems in structural mechanics. Vinus, 1968
- [3] Pochtman Yu. M., Hariton L.E. "Optimal design of constructions with regard to reliability Stroitel'naya mekhanika I raschet Sooruzheniy, 1976, № 6.
- [4] Bolotin V.V., "Statistical aspects in theory of structural stability" In dynamic stability of structures, Oxford, Pergamon Press, 1967.
- [5] Hotsialov N.F., "Safety factor" Stroitel'naya promyshlennost, 1929, № 10.
- [6] Timashev S.A., "The methods of construction reliability normalization with uneconomical responsibility" Reliability problems of reinforced concrete constructions Kuibyshev, 1973.
- [7] Bolotin V.V., "On combination of random loads acting on the construction, Stroitel'naya mekhanika i raschet sooruzheniy" 1962.
- [8] Bolotin V.V., "Problems of reliability theory in deformable solids mechanics", Continuum mechanics and related problems of analysis. M. Nauka, 1972.
- [9] Vasenin V.L., Dneprov I.V., "Sinyashek M.N. Reliability prediction of Martov type systems" Izvestiya Vuzov. Mashinostroyenie. 1977.
- [10] Bolotin V.V., "The methods of probability theory and reliability theory in construction analysis" M. 1982.
- [11] Venstel E.S., "Probability theory" Nauka, 1964.
- [12] Gnedenko B.V., "The course of probabilities" Fizmatyiz, 1961.
- [13] Rzhantsin A.R., "Statistical method of definition of admissible stresses under buckling" M. Stroyizdat, 1951.
- [14] Streletskiy N.S. "Up-to-date state of analysis of constructions" Izvestiya Vuzov. Stroitelstvo I architecture, 1963.
- [15] Building codes and rules. 2.03.01-84. Concrete and reinforced concrete constructions. M. Gosstroy SSSR, 1992.