Esnek Çoklu Topoloji ve Bazı Özellikleri Üzerine

Deniz Tokat[1,*], İsmail Osmanoğlu

[1]*Nevşehir Üniversitesi, Fen Edebiyat Fakültesi, Matematik Bölümü, Nevşehir*

Özet

Bu çalışma kapsamında Esnek Çoklu Küme, Esnek Topoloji ve bu topolojinin bazı temel özellikleri kullanılarak Esnek Çoklu Topoloji tanımlanmış ve bazı topolojik özellikleri belirtilmiştir.

Anahtar Kelimeler: Esnek küme, esnek çoklu küme, esnek topoloji, esnek çoklu topoloji

On the Soft Multi Topology and It's Some Properties

Abstract

In this article, Soft Multi Topology and some basic properties of this topology is defined by using Soft Multi Set, Soft Topology and some basic properties of this topology.

Keywords: Soft set, soft multi set, soft topology, soft multi topology

1.   Introduction

There are many uncertainties in most of the engineering, physics, computer sciences, economics, social sciences and medical sciences problems. Soft set theory was introduced by Molodtsov [1] as a mathematical tool for dealing with these uncertainties. He applied soft set theory in the first work successfully in many fields, such as smoothness of a function, game theory, the Riemann integral, Perron integral and measurement theory.

In addition to Molodtsov's works, soft set theory has applications in other areas, and the problems encountered in real life. Maji et al. [2, 3] presented an application of soft sets in decision making problems that are based on the reduction of parameters to keep the optimal choice objects. Chen [4] presented a new definition of soft set parameterization reduction and a comparison of it with attribute reduction in rough set theory. Pei and Miao [5] showed that soft sets are a class of special information systems. The application of soft set theory in algebraic structures was introduced by Aktaş and Çağman [6]. They discussed the notion of soft groups and derived some basic properties. Shabir and Naz [7] defined the soft topological space and studied the concepts of soft open set, soft interior point, soft neighbourhood of a point, soft separation axioms and subspace of a soft topological space. Aygünoğlu and Aygün [8] introduced the soft continuity of soft mapping, soft product topology and studied soft

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[*]e-mail: dtokat@nevsehir.edu.tr

compactness and generalized Tychonoff theorem to the soft topological space. Min [9] gave some results on soft topological spaces. Zorlutuna et al. [10] also investigated soft interior point and soft neighbourhood.

In classical set theory, there is no repetition of the set members. However, in some cases, repetition of element of set may be helpful. This set is called a multi set which is a collection of objects in which repetition of elements is significant. It has found many applications in real life in various fields like medicine, banking, engineering, information analysis, data analysis, data mining etc.

Multi set theory was introduced by Cerf et al. [11] in 1971. Peterson [12] and Yager [13] made further contributions to it. Many conclusive results were established by these authors and further study was carried on by Jena et al. [14]. Manjunath and John [15] have done some preliminary work on multi set relations. Following this study, multi set relation and multi set function was introduced by Girish and John [16]. In addition, these authors [17] using the multi sets relations gave multi set topology and some of the definitions of topological structures of this topology. Combining the concepts of soft set and multi set was given by Babitha and John [18].

In this article, we worked on how to construct soft multi topology, using properties of soft set, multi set and soft multi set.

2.   Preliminaries
In this section, we will give basic definitions of soft set and multi sets.

2.1.   Soft Set

Definition 2.1.1. ([1]) Let U be an initial universe set and E be set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq U$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F: A \rightarrow P(U)$.

Definition 2.1.2. ([3]) For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if

    i. $A \subseteq B$, and

    ii. $\forall \varepsilon \in A, F(\varepsilon)$ is subset of $G(\varepsilon)$.

We write $(F, A) \widetilde{\subseteq} (G, B)$.

Definition 2.1.3. ([3]) Soft sets (F, A) and (G, B) over a common universe U are said to be equal if (F, A) is soft multi subset of (G, B) and (G, B) is soft multi subset of (F, A).

Definition 2.1.4. ([3]) Let $A = \{e_1, e_2, \ldots, e_n\}$ be a set of parameters. The NOT set of A denoted by $]A$ is defined by $]A = \{ \,]e_1, \,]e_2, \ldots, \,]e_n\}$, where $\,]e_i = \text{not } e_i, \forall \, i = 1,2, \ldots, n$.

Proposition 2.1.5. ([3]) If A and B are two sets of parameters then we have the following

    i. $](]A) = A$

    ii. $](A \cup B) =]A \cup]B$

    iii. $](A \cap B) =]A \cap]B$

Definition 2.1.6. ([3]) The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c,]A)$ where $F^c:]A \rightarrow P(U)$ is a mapping given by $F^c(\alpha) = U - F(\alpha), \forall \alpha \in A$.

Let us call F^c to be the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F,A)^c)^c = (F,A)$.

Definition 2.1.7. ([3]) A soft set (F,A) over U is said to be absolute soft set denoted by $\tilde{A}$ if for all $\alpha \in A, F(\alpha) = U$.

Definition 2.1.8. ([3]) A soft set (F,A) over U is said to be null soft set denoted by Φ if for all $\alpha \in A, F(\alpha) = \emptyset$.

Definition 2.1.9. ([3]) If (F,A) and (G,B) over a common universe U are soft sets then (F,A) AND (G,B) denoted by $(F,A) \wedge (G,B)$ is defined as $(F,A) \wedge (G,B) = (H, A \times B)$ where $H(a,b) = F(a) \cap G(b)$. Here $\cap$ denotes the intersection of two sets $F(a)$ and $G(b)$.

Definition 2.1.10. ([3]) If (F,A) and (G,B) over a common universe U are soft sets then (F,A) OR (G,B) denoted by $(F,A) \vee (G,B)$ is defined as $(F,A) \vee (G,B) = (H, A \times B)$ where $H(a,b) = F(a) \cup G(b)$. Here $\cup$ denotes the union of two sets $F(a)$ and $G(b)$.

Definition 2.1.11. ([3]) Intersection of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = A \cap B$, and $\forall\, e \in C$, $H(e) = F(e) \cap G(e)$

We write $(F,A) \,\tilde{\cap}\, (G,B)$.

Definition 2.1.12. ([3]) Union of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and $\forall\, e \in C$

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

We write $(F,A) \,\tilde{\cup}\, (G,B)$.

Definition 2.1.13. ([7]) The difference (H,E) of two soft sets (F,E) and (G,E) over X, denoted by $(F,E) \setminus (G,E)$, is defined as $H(e) = F(e) \setminus G(e)$ for all $e \in E$.

Definition 2.1.14. ([7]) Let (F,E) be a soft set over X and $x \in X$. We say that $x \in (F,E)$ read as x belongs to the soft set (F,E)

whenever $x \in F(\alpha)$ for all $\alpha \in E$.

Note that for any $x \in X, x \notin (F,E)$, if $x \notin F(\alpha)$ for some $\alpha \in E$.

Definition 2.1.15. ([7]) Let (F,E) be a soft set over X and Y be a non-empty subset of X. Then the sub soft set of (F,E) over Y denoted by $(^Y F, E)$, is defined as follows $^Y F(\alpha) = Y \cap F(\alpha)$, for all $\alpha \in E$. In other words $(^Y F, E) = \tilde{Y} \cap (F,E)$.

Definition 2.1.16. ([7]) The relative complement of a soft set (F,A) is denoted by $(F,A)'$ and is defined by $(F,A)' = (F',A)$ where $F' : A \to P(U)$ is a mapping given by $F'(\alpha) = U - F(\alpha)$ for all $\alpha \in A$.

Proposition 2.1.17. ([7]) Let (F,E) and (G,E) be the soft sets over X. Then

     (1) $((F,E) \cup (G,E))' = (F,E)' \cap (G,E)'$,

     (2) $((F,E) \cap (G,E))' = (F,E)' \cup (G,E)'$.

2.2. Multi Set

Definition 2.2.1. ([1]) An mset M drawn from the set X is represented by a function Count M or C_M defined as $C_M : X \to N$ where N represented the set of non negative integers. The word "multi set" often shortened to "mset".

Definition 2.2.2. ([1]) Let M_1 and M_2 be two msets drawn from a set X. An mset M_1 is a submset of M_2 if $C_{M_1}(x) \leq C_{M_2}(x)$ for all $x \in X$.

Definition 2.2.3. ([1]) The union of two msets M_1 and M_2 drawn from a set X is a mset M denoted by $M = M_1 \cup M_2$ such that $x \in X, C_M(x) = \max\{C_{M_1}(x), C_{M_2}(x)\}$.

Definition 2.2.4. ([1]) The intersection of two msets M_1 and M_2 drawn from a set X is a mset M denoted by $M = M_1 \cap M_2$ such that $x \in X, C_M(x) = \min\{C_{M_1}(x), C_{M_2}(x)\}$.

Definition 2.2.5. ([7]) A submset N of M is a whole submset of M with each element in N having full multiplicity as in M. i.e., $C_N(x) = C_M(x)$ for every x in N.

Definition 2.2.6. ([1]) Let $[X]^m$ denotes the set of all mset whose elements are in X such that no element in mset occurs more than m times. Let $M \in [X]^m$ be an mset. The power whole mset of M denoted PW(M) is defined as the set of all whole submset of M. i.e., for constructing power whole submset of M, every element of M with its full multiplicity behaves like an element in a classical set. The cardinality of PW(M) is 2^n where n is the cardinality of the support set (root set) of M.

Notation 2.2.7. Let M be an mset from X with x appearing n times in M. It is denoted by $x \in^n M. M = \{k_1/x_1, k_2/x_2, .., k_n/x_n\}$ where M is an mset with x_1 appearing k_1 times, x_2 appearing k_2 times, and so on.

3. Soft Topology

Definition 3.1. ([7]) Let T be the collection of soft sets over X, then T is said to be a soft topology on X if

(1) $\Phi, \tilde{X}$ belong to T

(2) the union of any number of soft sets in T belongs to T

(3) the intersection of any two soft sets in T belongs to T .

The triple (X, T, E) is called a soft topological space over X.

Definition 3.2. ([19]) Let (X, T_1, E) and (X, T_2, E) be soft topological spaces. Then, the following hold.

If $T_2 \supseteq T_1$, then T_2 is soft finer than T_1.

If $T_2 \supset T_1$, then T_2 is soft strictly finer than T_1.

If either $T_2 \supseteq T_1$ or $T_2 \subseteq T_1$, then T_1 is comparable with T_2.

Definition 3.3. ([7]) Let (X, T, E) be a soft space over X, then the members of T are said to be soft open sets in X.

Definition 3.4. ([7]) Let (X, T, E) be a soft space over X. A soft set (F, E) over X is said to be a soft closed set in X, if its relative complement $(F, E)'$ belongs to T .

Definition 3.5. ([7]) Let X be an initial universe set, E be the set of parameters and $T = \{\Phi, \tilde{X}\}$. Then T is called the soft indiscrete topology on X and (X, T, E) is said to be a soft indiscrete space over X.

Definition 3.6. ([7]) Let X be an initial universe set, E be the set of parameters and let T be the collection of all soft sets which can be defined over X. Then T is called the soft discrete topology on X and (X, T, E) is said to be a soft discrete space over X.

Proposition 3.7. ([7]) Let (X, T, E) be a soft space over X. Then the collection $T_\alpha = \{F(\alpha) \mid (F, E) \in T\}$ for each $\alpha \in E$, defines a topology on X.

Example 3.8. Let $X = \{a, b, c, d\}, E = \{e_1, e_2\}$ and $T = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ are soft sets over X, defined as follows

$$F_1(e_1) = \{a\}, \qquad F_1(e_2) = \{d\}$$
$$F_2(e_1) = \{a, b\}, \qquad F_2(e_2) = \{b, d\}$$
$$F_3(e_1) = \{a, c, d\}, \quad F_3(e_2) = \{a, c, d\}$$

Then T defines a soft topology on X and hence (X, T, E) is a soft topological space over X. It can be easily seen that

$$T_{e_1} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}\}$$

and

$$T_{e_2} = \{\emptyset, X, \{d\}, \{b, d\}, \{a, c, d\}\}$$

are topologies on X.

Proposition 3.9. ([7]) Let (X, T_1, E) and (X, T_2, E) be two soft topological spaces over the same universe X,

then $(X, T_1 \cap T_2, E)$ is a soft topological space over X.

Definition 3.10. ([7]) Let (X, T, E) be a soft topological space over X and (F, E) be a soft set over X. Then the soft closure of (F, E), denoted by $\overline{(F, E)}$ is the intersection of all soft closed super sets of (F, E). Clearly $\overline{(F, E)}$ is the smallest soft closed set over X which contains (F, E).

Definition 3.11. ([7]) Let (X, T, E) be a soft topological space over X, $x \in X$ be a soft set over X and $x \in X$. Then x is said to be a soft interior point of (G, E) if there exists a soft open set (F, E) such that $x \in (F, E) \widetilde{\subseteq} (G, E)$.

Definition 3.12. ([7]) Let (X, T, E) be a soft topological space over X, (G, E) be a soft set over X and $x \in X$. Then (G, E) is said to be a soft neighbourhood of x if there exists a soft open set (F, E) such that $x \in (F, E) \widetilde{\subseteq} (G, E)$.

Definition 3.13. ([19]) Let (X, T, E) be a soft topological space, $(F, A) \widetilde{\subseteq} \widetilde{X}$, and $\alpha \in \widetilde{X}$ If every soft neighborhood of α intersects (F, A) in some points other than α itself, then α is called a soft limit point of (F, A).

In other words, if (X, T, E) is a soft topological space, $(F, A), (F, B) \widetilde{\subseteq} \widetilde{X}$, and $\alpha \in \widetilde{X}$, then α is a soft limit point of $(F, A) \Leftrightarrow (F, B) \widetilde{\cap} \left((F, A) \backslash \{\alpha\}\right) \neq \Phi$ for every soft neighbourhood of α.

Definition 3.14. ([7]) Let (X, T, E) be a soft topological space over X and Y be a non-empty subset of X. Then

$$T_Y = \{(\,^Y F, E) \mid (F, E) \in T\}$$

is said to be the soft relative topology on Y and (Y, T_Y, E) is called a soft subspace of (X, T, E).

We can easily verify that T_Y is, in fact, a soft topology on Y.

Example 3.15. Any soft subspace of a soft discrete topological space is a soft discrete topological space.

Example 3.16. Any soft subspace of a soft indiscrete topological space is a soft indiscrete topological space.

4. Soft Multi Sets and Soft Multi Topology

4.1. Soft Multi Sets

Definition 4.1.1. ([18]) Let U be universal mset and E be set of parameters. Then an ordered pair (F, E) is called a soft multi set where F is a mapping given by $F: A \to PW(U)$

Example 4.1.2. Let U be universal mset consist of balls under consideration.

$U = \{k_1/b_1, \ k_2/b_2, k_3/b_3, k_4/b_4, k_5/b_5, k_6/b_6\}$ where k_i denotes the multiplicity of ball b_i. Let $A = \{black, red, blue\}$. Then the soft multi set (F, A) defined below gives different colors of balls under consideration.

$$F(black) \ = \ \{k_2/b_2, k_3/b_3\}$$
$$F(red) \ = \ \{k_1/b_1, k_4/b_4\}$$
$$F(blue) \ = \ \{k_5/b_5, k_6/b_6\}$$

Note that for the approximations "blackballs" $= \{k_2/b_2, k_3/b_3\}$ the multiplicity of element is same as those of the universal mset. Here the approximation set is a multiset.

Definition 4.1.3. ([18]) For two soft multi sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft multi subset of (G, B) if

      i. $A \subseteq B$, and

      ii. $\forall \varepsilon \in A, F(\varepsilon)$ is multi subset of $G(\varepsilon)$.

We write $(F, A) \ \widetilde{\subseteq} \ (G, B)$.

Definition 4.1.4. ([18]) Soft multi sets (F, A) and (G, B) over a common universe U are said to be equal if (F, A) is soft multi subset of (G, B) and (G, B) is soft multi subset of (F, A).

Definition 4.1.5. ([18]) Let M be a multi set. Then the relative compliment of a whole submset M_1 of M is given by $M_1^r = m_i/x_i$ where $C_M(x_i) = 0$ for every x_i in M_1 and m_i is the count of x_i in M.

Example 4.1.6. Let

$$M \ = \ \{k_1/x_1, k_2/x_2, k_3/x_3, k_4/x_4, k_5/x_5\}$$
$$M_1 \ = \ \{k_2/x_2, k_4/x_4, k_5/x_5\}$$

Then the relative complement of M_1 is given by $M_1^r = \{k_1/x_1, k_3/x_3\}$

In the following definition the complement of soft multi set is taken as relative complement.

Definition 4.1.7. ([18]) The complement of a soft multi set $F, A)$ (is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \rceil A)$ where $F^c(\rceil\alpha) = F^r(\alpha)$ for every $\rceil\alpha \in \rceil A$.

Definition 4.1.8. ([18]) A soft multi set (F, A) over universe U is said to be absolute soft multiset denoted by $\widetilde{A}$ if for all $\alpha \in A, F(\alpha) = U$.

Definition 4.1.9. ([18]) A soft multi set (F, A) over universe U is said to be null soft multiset denoted by Φ if for all $\alpha \in A, F(\alpha) = \emptyset$.

Definition 4.1.10. ([18]) If (F, A) and (G, B) over a common universe U are soft multi sets then (F, A) AND (G, B) denoted by $(F, A) \wedge (G, B)$ is defined as $(F, A) \wedge (G, B) = (H, A \times B)$ where $H(a, b) = F(a) \cap G(b)$. Here $\cap$ denotes the intersection of two multi sets $F(a)$ and $G(b)$.

Definition 4.1.11. ([18]) If (F, A) and (G, B) over a common universe U are soft multisets then (F, A) OR (G, B) denoted by $(F, A) \vee (G, B)$ is defined as $(F, A) \vee (G, B) = (H, A \times B)$ where $H(a, b) = F(a) \cup G(b)$. Here $\cap$ denotes the union of two multisets $F(a)$ and $G(b)$.

Definition 4.1.12. ([18]) Intersection of two soft multi sets (F, A) and (G, B) over a common universe U is the soft multi set (H, C), where $C = A \cup B$, and $\forall e \in C$

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cap G(e), & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A) \, \widetilde{\cap} \, (G, B)$.

Definition 4.1.13. ([18]) Union of two soft multi sets (F, A) and (G, B) over a common universe U is the soft multi set (H, C), where $C = A \cup B$, and $\forall \, e \in C$

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A) \, \widetilde{\cup} \, (G, B)$.

4.2. Soft Multi Topology

Definition 4.2.1. Let T be the collection of soft multi sets over X, then T is said to be a soft multi topology on X if

(1) $\Phi, \widetilde{X}$ belong to T

(2) the union of any number of soft multi sets in T belongs to T

(3) the intersection of any two soft multi sets in T belongs to T .

The triple (X, T, E) is called a soft multi topological space over X.

Definition 4.2.2. Let (X, T_1, E) and (X, T_2, E) be soft multi topological spaces. Then, the following hold.

If $T_2 \supseteq T_1$, then T_2 is soft finer than T_1.

If $T_2 \supset T_1$, then T_2 is soft strictly finer than T_1.

If either $T_2 \supseteq T_1$ or $T_2 \subseteq T_1$, then T_1 is comparable with T_2.

Definition 4.2.3. Let (X, T, E) be a soft multi space over X, then the members of T are said to be soft multi open sets in X.

Definition 4.2.4. Let (X, T, E) be a soft multi space over X. A soft set (F, E) over X is said to be a soft multi closed set in X, if its relative complement $(F, E)'$ belongs to T .

Definition 4.2.5. Let X be an initial universe multi set, E be the set of parameters and $T = \{\Phi, \widetilde{X}\}$. Then T is called the soft multi indiscrete topology on X and (X, T, E) is said to be a soft indiscrete space over X.

Definition 4.2.6. Let X be an initial universe multi set, E be the set of parameters and let T be the collection of all soft multi sets which can be defined over X. Then T is called the soft multi discrete topology on X and (X, T, E) is said to be a soft multi discrete space over X.

Proposition 4.2.7. Let (X, T, E) be a soft multi space over X. Then the collection $T_\alpha = \{F(\alpha) \mid (F, E) \in T\}$ for each $\alpha \in E$, defines a topology on X.

Proof. By definition, for any $\alpha \in E$, we have $T_\alpha = \{F(\alpha) \mid (F, E) \in T\}$. Now,

(1) $\Phi, \widetilde{X} \in T$ implies that $\emptyset, X \in T_\alpha$.

(2) Let $\{F_i(\alpha) \mid i \in I\}$ be a collection of sets in T_α. Since $(F_i, E) \in T$, for all $i \in I$ so that $\cup_{i \in I} (F_i, E) \in T$ thus $\cup_{i \in I} F_i(\alpha) \in T_\alpha$.

(3) Let $F(\alpha), G(\alpha) \in T_\alpha$ for some $(F, E), (G, E) \in T$. Since $(F, E) \cap (G, E) \in T$ so $F(\alpha) \cap G(\alpha) \in T_\alpha$.

Thus T_α defines a topology on X for each $\alpha \in E$.

Example 4.2.8. Let $X = \{2/a, 1/b, 3/c, 2/d\}, E = \{e_1, e_2\}$ and $T = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ where $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ are soft multi sets over X, defined as follows

$$F_1(e_1) = \{2/a\}, \qquad F_1(e_2) = \{1/b\},$$
$$F_2(e_1) = \{1/b\}, \qquad F_2(e_2) = \{2/a, 3/c, 2/d\},$$

$$F_3(e_1) = \{2/a, 1/b\}, \qquad F_3(e_2) = X,$$
$$F_4(e_1) = \{1/b, 3/c, 2/d\}, \quad F_4(e_1) = \{2/a, 3/c, 2/d\}.$$

Then T defines a soft multi topology on X and hence (X, T, E) is a soft multi topological space over X. It can be easily seen that

$$T_{e_1} = \{\emptyset, X, \{2/a\}, \{1/b\}, \{2/a, 1/b\}, \{1/b, 3/c, 2/d\}\}$$

and

$$T_{e_2} = \{\emptyset, X, \{1/b\}, \{2/a\}, \{2/a, 3/c, ,2/d\}\}$$

are topologies on X.

Proposition 4.2.9. Let (X, T_1, E) and (X, T_2, E) be two soft multi topological spaces over the same universe X,

then $(X, T_1 \cap T_2, E)$ is a soft multi topological space over X.

Proof. (1) $\Phi, \widetilde{X}$ belong to $T_1 \cap T_2$.

(2) Let $\{(F_i, E) \mid i \in I\}$ be a family of soft multi sets in $T_1 \cap T_2$. Then $(F_i, E) \in T_1$ and $(F_i, E) \in T_2$, for all $i \in I$, so $\cup_{i \in I} (F_i, E) \in T_1$ and $\cup_{i \in I} (F_i, E) \in T_2$. Thus $\cup_{i \in I} (F_i, E) \in T_1 \cap T_2$.

(3) Let $(F, E), (G, E) \in T_1 \cap T_2$. Then $(F, E), (G, E) \in T_1$ and $(F, E), (G, E) \in T_2$. $(F, E) \cap (G, E) \in T_1$ and $(F, E) \cap (G, E) \in T_2$, so $(F, E) \cap (G, E) \in T_1 \cap T_2$.

Thus $T_1 \cap T_2$ defines a soft multi topology on X and $(X, T_1 \cap T_2, E)$ is a soft multi topological space over X.

Definition 4.2.10. Let (X, T, E) be a soft multi topological space over X and (F, E) be a soft multi set over X. Then the soft multi closure of (F, E), denoted by $\overline{(F, E)}$ is the intersection of all soft multi closed super sets of (F, E).

Clearly $\overline{(F, E)}$ is the smallest soft multi closed set over X which contains (F, E).

Definition 4.2.11. Let (X, T, E) be a soft multi topological space over X, $x \in X$ be a soft multi set over X and $x \in X$. Then x is said to be a soft interior point of (G, E) if there exists a soft multi open set (F, E) such that $x \in (F, E) \widetilde{\subseteq} (G, E)$. Set of all soft multi interior point of (G, E) is called the soft multi interior of (G, E) and denoted by $(G, E)^\circ$.

Definition 4.2.12. Let (X, T, E) be a soft multi topological space over X, (G, E) be a soft multi set over X and $x \in X$. Then (G, E) is said to be a soft multi neighborhood of x if there exists a soft multi open set (F, E) such that $x \in (F, E) \widetilde{\subseteq} (G, E)$. The set of all soft neighborhoods of α, denoted $\widetilde{N}(\alpha)$, is called the family of soft multi neighborhoods of α; that is,

$$\widetilde{N}(\alpha) = \{(G, E) : (G, E) \in T, \alpha \in (G, E)\}.$$

Definition 4.2.12. Let (X, T, E) be a soft multi topological space, $(F, A) \widetilde{\subseteq} \widetilde{X}$, and $\alpha \in \widetilde{X}$ If every soft multi neighborhood of α intersects (F, A) in some points other than α itself, then α is called a soft multi limit point of (F, A).

In other words, if (X, T, E) is a soft multi topological space, $(F, A), (F, B) \widetilde{\subseteq} \widetilde{X}$, and $\alpha \in \widetilde{X}$, then α is a soft multi limit point of $(F, A) \Leftrightarrow (F, B) \widetilde{\cap} ((F, A)\backslash\{\alpha\}) \neq \Phi$ for every $(F, A) \in \widetilde{N}(\alpha)$.

Example 4.2.13. Let us consider $T = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ soft multi topology given in Example 4.2.8. we obtain respectively;

$$\overline{(F_1, E)} = (F_2, E)^c \cap (F_4, E)^c = (F_1, E),$$

$$\tilde{N}(a) = \{(F_3, E)\},$$
$$\tilde{N}(b) = \{(F_3, E)\},$$
$$(F_3, E)^\circ = \{2/a, 1/b\}.$$

Definition 4.2.14. Let (X, T, E) be a soft multi topological space over X and Y be a non-empty subset of X. Then

$$T_Y = \{(\ ^Y F, E) \mid (F, E) \in T \}$$

is said to be the soft multi relative topology on Y and (Y, T_Y, E) is called a soft multi subspace of (X, T, E). We can easily verify that T_Y is, in fact, a soft multi topology on Y.

Example 4.2.15. Any soft multi subspace of a soft discrete topological space is a soft multi discrete topological space.

Example 4.2.16. Any soft multi subspace of a soft indiscrete topological space is a soft multi indiscrete topological space.

5. Conclusion

In this paper, we introduced the notion of soft multi topological space and defined basic concepts such as soft multi open set, soft multi closed set, soft multi closure, soft multi interior, soft multi neighborhood and limit point of a soft multi set. Moreover, soft multi subspace and soft multi discrete and soft multi indiscrete topologies are given. This paper may be the starting point for the soft multi topology and its application to dealing with decision making problems.

6. References

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